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16. ABSTRACT

Procedures for evaluating the technical, economic, and financial aspects of urban water quality management planning problems are presented. Accepted principles of benefit-cost analysis are used to conduct the economic analysis. Benefits are measured as the reduction in damages as controls are employed. A case by case procedure for examining wet-weather impacts is recommended. Production functions which provide a summary characterization of the performance of the control units, are presented for a relatively wide variety of control options. The economic optimization procedures utilize the production functions along with associated cost data to find the most cost-effective solution. The financial analysis combines existing procedures with new concepts from N person game theory to provide improved methods for solving this important problem. The interrelationship between the economic and financial analysis is demonstrated. The problem of finding the fairest way to assign costs in a multi-purpose and/or multi-group problem depends partially on circumstances unique to each problem. Then, several case studies are presented to illustrate the procedure.

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by

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FOREWORD

The U.S. Environmental Protection Agency was created because of increasing public and government concern about the dangers of pollution to the health and welfare of the American people. Noxious air, foul water, and spoiled land are tragic testimony to the deterioration of our natural environment. The complexity of that environment and the interplay between its components require a concentrated and integrated attack on the problem.

Research and development is that necessary first step in problem solution and it involves defining the problem, measuring its impact, and searching for solutions. The Municipal Environmental Research Laboratory develops new and improved technology and systems for the prevention, treatment, and management of wastewater and solid and hazardous waste pollutant discharges from municipal and community sources, for the preservation and treatment of public drinking water supplies and to minimize the adverse economic, social, health, and aesthetic effects of pollution. This publication is one of the products of that research; a most vital communications link between the researcher and the user community.

The ever increasing competition for public money makes it essential that the cost effectiveness of water pollution control programs be evaluated. This report presents procedures for evaluating a wide variety of investment alternatives for controlling wastewater and stormwater runoff. The primary justification for these programs is protection of public health. Various procedures for evaluating public health benefits are described. Then, a wide variety of cost-effectiveness problems are solved ranging from simply selecting the best wastewater treatment plant to a much more complicated analysis of traditional waste treatment, and best management practices options within a multi-purpose, multi-group decision setting. The fairness of proposed solutions is evaluated by combining accepted cost allocation/ sharing arrangements with concepts from cooperative N-person game theory. The resulting procedure is a much more general method for assessing the fairness of the proposed cost-effective solution. Numerous examples are included.

Francis T. Mayo, Director
Municipal Environmental Research Laboratory

PREFACE

This report is part of a series of documents on urban stormwater management which provides a variety of tools for evaluating alternatives ranging from simple desktop procedures (Level I analysis) to sophisticated computer-based simulation using the original EPA Storm Water Management Model (Level IV analysis). Other documents in this series are:

Metcalf and Eddy, Inc., University of Florida, and Water Resources Engineers, Inc., "Storm Water Management Model, Volume I - Final Report, "EPA Report No. 11024 DOC 07/71, (NTIS PB 203 289), July 1971.

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Heaney, J.P., Huber, W.C., Medina, M.A., Jr., Murphy, M.P., Nix, S.J. and Hasan, S., "Nationwide Evaluation of Combined Sewer Overflows and Urban Stormwater Discharges, Volume II: Cost Assessment and Impacts," EPA Report No. EPA-600/2-77-064b, (NTIS PB 266 005), March 1977.

Heaney, J.P. and Nix, S.J., "Storm Water Management Model: Level I - Comparative Evaluation of Storage-Treatment and Other Management Practices," EPA Report No. EPA-600/2-77-083, (NTIS PB 265 671), April 1977.

Medina, M.A., "Receiving Water Quality Modeling for Urban Stormwater Management: Level III," EPA Report No. EPA-600/2-79-100, August 1979.

Huber, W.C., Heaney, J.P. and Nix, S.J., "Storm Water Management Model User's Manual - Version III," EPA Draft Report, EPA Grant No. R805664 and R802411, 1980.

ABSTRACT

Procedures for evaluating the technical, economic, and financial aspects of urban water quality management planning problems are presented. Accepted principles of benefit-cost analysis are used to conduct the economic analysis which takes advantage of opportunities to save money arising from constructing larger control units and/or from more fully utilizing the assimilative capacity of the receiving water. Benefits are measured as the reduction in damages as controls are employed. Various indirect and direct measures of damages are presented. A case by case procedure for examining wet-weather impacts is recommended.

The technical aspects of the problem are presented as production functions which provide a summary characterization of the performance of the control units. The need to develop these production functions for the various control processes using a consistent set of assumptions is emphasized. Functions are presented for a relatively wide variety of control options. The economic optimization procedures utilize the production functions along with associated cost data to find the most cost-effective solution. Several problems of increasing complexity are examined.

The financial analysis combines existing procedures with new concepts from N-person game theory to provide improved methods for solving this important problem. The interrelationship between the economic and financial analysis is demonstrated. The problem of finding the fairest way to assign costs in a multi-purpose and/or multi-group problem depends partially on circumstances unique to each problem. Thus, several case studies are presented to illustrate the procedure.

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LIST OF ABBREVIATIONS AND SYMBOLS

ABBREVIATIONS

AJE -- Alternative Justifiable Expenditures

AP -- average productivity

AR -- annual runoff

BMP -- best management practice

BOD_5 -- five day biochemical oxygen demand

BPT -- best practicable treatment

CA -- amortized capital costs

COD -- chemical oxygen demand

DO -- dissolved oxygen

DWF -- dry-weather flow

EB -- excess benefits

EBC -- minimum of excess benefits or costs

EC -- excess costs

EPA -- Environmental Protection Agency

EQ -- environmental quality

FC -- fixed cost

$FeCl_3$ -- ferric chloride

FWPCA -- Federal Water Pollution Control Administration

GID -- Grainlands Irrigation District

LC_x -- lethal concentration to kill x per cent of the population

MB -- marginal benefits

ABBREVIATIONS AND SYMBOLS (CONTINUED)

MC -- marginal costs

MCRS -- minimum costs, remaining savings method

mgd -- million gallons per day

mg/ℓ -- milligrams per liter

MLVSS -- mixed liquor volatile suspended solids

MP -- marginal productivity

MPSX -- extended version of IBM's mathematical programming system computer code

MRTS -- marginal rate of technical substitution

NCWQ -- National Commission on Water Quality

NH_3-N -- ammonia nitrogen

NPS -- nonpoint source

NSC -- nonseparable cost

OID -- Orchard Irrigation District

OM -- operation and maintenance costs

P -- total phosphorus

PRM -- Program Requirements Memorandum

RC -- remaining costs

SC -- separable costs

SCRB -- separable cost, remaining benefits method

STORM -- Storage, Treatment, Overflow, and Runoff Model

STP -- sewage treatment plant

SWMM -- Storm Water Management Model

TC -- total cost

TSS -- total suspended solids

ABBREVIATIONS AND SYMBOLS (CONTINUED)

TVA -- Tennessee Valley Authority

UOD -- ultimate oxygen demand

VC -- variable cost

WWF -- wet-weather flow

7Q10 -- once-in-ten-year seven-day low flow

SYMBOLS

a -- Coefficient

A -- Coefficient

α -- Weighting factor

b -- Coefficient

B -- Benefits

β -- Coefficient

c -- Unit cost

C -- Capital

d -- Coefficient

e -- minimum savings

H -- Herd size

k -- constant

k_i -- Proportion of population killed by i^{th} event

L -- Labor

L -- Pipeline length

m -- Coefficient

MR -- Marginal return per cow

ABBREVIATIONS AND SYMBOLS (CONTINUED)

n -- number of groups
N_i -- Population after i^{th} killing event
N_o -- Initial population
N_n -- Final population
P -- Manufacturing output
Q -- Flow rate
r -- Average rate of return per cow
r_i -- Sum of groups of players
s -- Annual soil loss coefficient
S -- Storage volume
t -- Detention time
T -- Treatment rate
v -- value
V -- Tank volume
x_1 -- Input 1
x_2 -- Input 2
$\bar{X}$ -- Vector of inputs
y -- Output
$\bar{Y}$ -- Vector of outputs
Z -- Value of the objective function

Ψ -- Level of treatment for a single facility
ϕ -- Level of treatment for a joint facility
γ -- Coefficient
η -- Removal efficiency

ABBREVIATIONS AND SYMBOLS (CONCLUDED)

ϕ -- empty set

θ -- minimum value needed to obtain a nonempty core

CONVERSION FACTORS
ENGLISH TO METRIC UNITS

English Unit	Abbr.	Multiplier	Abbr.	Metric Unit
Acre	acre	0.405	ha	hectare
Foot (feet)	ft	0.305	m	meter(s)
Gallon(s)	gal	3.785	l	liter(s)
		3.785×10^{-3}	m^3	cubic meter(s)
Gallons per day per square foot	gpd/ft^2	0.041	$m^3/day/m^2$	cubic meters per day per square meter
Gallons per minute per square foot	gpm/ft^2	2.45	$m^3/hr/m^2$	cub meters per hour per square meter
Gallons per minute	gpm	0.0631	l/s	liters per second
Horsepower	hp	0.746	kW	kilowatts
Inch (es)	in.	2.54	cm	centimeter
Million gallons per day	mgd	3,785	m^3/day	cubic meters per day
Parts per billion	ppb	1.0	ug/l	micrograms per liter
Parts per million	ppm	1.0	mg/l	milligrams per liter
Pound (s)	lb	0.454	kg	kilogram
		453.6	g	grams
Pounds per cubic foot	lb/ft^3	16.02	kg/m^3	kilograms per cubic meter
Pounds per square foot	lb/ft^2	4.88	kg/m^2	kilograms per sq. meter
Pounds per square inch	psi	0.0703	kg/cm^2	kilograms per sq. centimeter
Square foot	ft^2	0.0929	m^2	Square center
Square inch	in.^{2}	6.452	cm^2	square centimeter
Standard cubic feet per minute	scfm	1.699	m^3/hr	cubic meter per hour
Standard cub feet per minute per sq. foot	scfm/ft^2	0.305	$m^3/min/m^2$	cubic meters per minute per square meter
Ton (short)	ton	907.2	kg	kilograms
		0.907	metric ton	metric ton
Yard	yd	0.914	m	meter

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CHAPTER 1

INTRODUCTION

This report presents procedures for evaluating the technical, economic, and financial aspects of urban water quality management planning problems. Institutional, political, and legal aspects are not discussed. The need for this type of evaluation procedure arises due to the requirements of the Federal Water Pollution Control Act Amendments of 1972 to go beyond the traditional evaluation of major continuous municipal and industrial sources, and to examine intermittent sources from combined sewer overflows and urban stormwater runoff. In addition, other sources such as construction activities, mining, and rural nonpoint runoff need to be examined.

These more complicated evaluations are being conducted as part of Section 208 of the 1972 legislation. New procedures are needed to permit analysts to make much more complicated evaluations. Accepted principles of benefit-cost analysis are used to perform the economic analysis. Background material on this subject is included to assist the reader. The solution to the economic analysis is the most cost-effective way of satisfying a prescribed water quality standard. The economic analysis takes advantage of all available opportunities to save money arising from constructing larger control units and/or from more fully utilizing the assimilative capacity of the receiving water. While the methods to be presented are relatively straight forward the derived solutions may not be too dependable unless extensive local data are available. The analysis is complicated by the intermittent nature of the storm flows, the lack of sufficient experience to adequately estimate the performance of control options, and the uncertainty regarding receiving water impacts. It is difficult to evaluate the receiving water impact of intermittent pollution sources in the presence of significant point sources or when residual benthal deposits from point sources still exert a predominant influence. Nonetheless, current EPA requirements mandate that benefits be estimated.

Some of the most promising opportunities for cost-effective environmental control are multi-purpose in nature. The second major part of this report presents procedures for fairly assigning the total cost of the project among purposes and/or groups. Reliable methods for doing this financial analysis do not exist. Thus, new procedures were devised which link accepted practice in water resources planning with the theoretical basis provided by cooperative N-person game theory. Briefly, the financial analysis examines the question of whether an assignment of costs can be made among the participants which leaves each one of them no worse off (hopefully better off) than they would be otherwise. The results from this analysis provide valuable insight as to the goodness of the economically optimal solution. In many cases, the savings are not significant and some of the participants may not feel that cooperation is worthwhile. Thus, the financial study provides a nice sensitivity analysis on the economic appraisal. More directly, in some cases, the cost functions used in the economic analysis depend on the cost sharing rule of the financial analysis. Thus, the two studies are interrelated.

The next three chapters deal with economic analysis within a benefit-cost analysis framework. Figure 1-1 shows the general procedure for finding the optimal level of pollution control. The pollution damages curve represents the amount of disbenefits that society suffers as a result of uncontrolled wastewater discharges. The assumed shape of the damage curve is very important. As controls are initiated, these monies are spent controlling the more flagrant problems, e.g., water-born epidemics. Thus, damages fall at a relatively rapid rate. As the level of pollution control increases beyond this initial phase, further controls yield progressively less reduction in damages. Thus, in principle, the curve will have the shape shown in Figure 1-1. The major weakness in this phase of the analysis is our inability to convert health, aesthetics, and related damages into a monetary equivalent.

The abscissa of Figure 1-1 is "% pollution control." BOD_5 has been the most commonly used surrogate for "pollution." However, more recently, attention has shifted to interest in nutrients and toxics. Thus, the related measure is an important part of the analysis. Chapter 3 describes procedures for quantifying damages and using various measures of effectiveness.

The shape of the cost curve shown in Figure 1-1 is also important. Lower levels of control are relatively less expensive since a wider variety of controls may be used to remove those forms of pollution which are easiest to control. However, control costs begin to increase relatively rapidly at higher levels of control as the limits of technology are approached. It is possible to obtain relatively good data on control costs. Chapter 4 describes a consistent procedure for characterizing control options as production functions. The shape of the production function has a direct influence on the shape of the cost functions.

To find the optimal level of pollution control, the damage and cost curves are added to obtain the total cost curve shown in Figure 1-1. For this example, the optimal control level, p^*, is about 53%. For any level of control less than 53% every dollar invested in pollution control reduces damages by more than one dollar. At 53%, the last dollar spent reduces damages by exactly one dollar. Every dollar spent beyond 53% control yields progressively less than one dollar in return. This procedure works very well if benefit and cost data are available. However, since it is very difficult to obtain benefit data for pollution control (as it is for most areas of public investment), a cost-effectiveness approach is used. The measure of effectiveness is usually a water quality standard. Given such a standard, Chapter 5 presents procedures for doing cost-effectiveness analysis for a relatively wide variety of water pollution problems.

The results of the economic analysis will typically indicate that the best overall solution is to use a blend of many controls. While this solu-

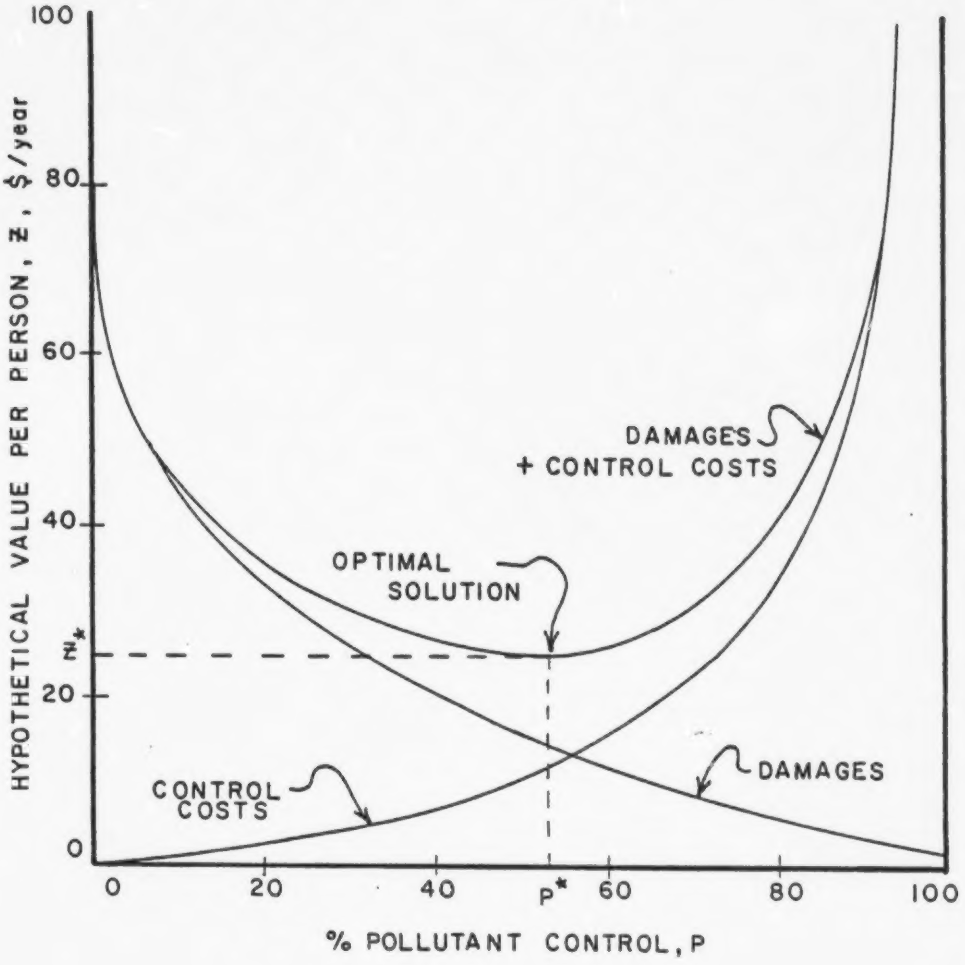

Figure 1-1. Optimal Level of Pollution Control

tion may be optimal, it may not be implementable unless a cost allocation (among purposes) and sharing (among groups) arrangement can be worked out. Existing methods for assigning costs do not always work, i.e., they lead to unfair assignment of costs. The economic analysis (Chapter 5) assumes unit costs based on reported values in the literature. These unit costs vary widely depending on the cost allocation and/or sharing procedures used. Unresolved theoretical problems exist in addressing questions as to the proper measure of costs to use. Techniques from game theory are combined with accepted practice to produce a more general procedure. The results are presented in Chapter 6.

The methods presented in Chapter 6 are not the ultimate answer to this problem. Case by case analysis is necessary because a good part of the definition of "fairness" is site specific. Also, many practictioners may be suspicious of the newer methods. Chapter 7 presents several case studies to demonstrate the procedure.

CHAPTER 2

CONCLUSIONS AND RECOMMENDATIONS

DAMAGE ASSESSMENT

The most obvious forms of pollution have been controlled following a decade of an aggressive national program. It is physically impossible to remove all of the remaining pollution. The critical question remains of how much of this residual should be controlled. An overall framework for water pollution policies indicates that this is just one of numerous ways of striving toward the broader societal goal of promotion of the general welfare. Within this framework, the desirability of continued expenditures in water pollution control can be compared with other investments to promote man's well being. Within this context, the value of natural systems is related to protecting man's well-being. Admittedly, this is an imperfect valuation system. However, it is essential to use a system of accounts that can be related directly to other expenditures. It is impossible to measure the relative merits of spending one million dollars to enhance ecosystem stability versus a like expenditure on public health clinics.

WATER QUALITY BENEFITS

Damage assessment is an increasingly difficult task because the remaining pollutants are present in lower concentrations and have less obvious impacts. Furthermore, there is disagreement on what constitutes a problem. Benefits are measured in terms of changes in pollution caused damages. Ideally, a monetary equivalent can be estimated but this is usually impossible since many of the damages are health related and cannot be assigned a simple monetary equivalent. Available studies indicate that per capita benefits from additional expenditures to enhance water-based recreational opportunities are about $10 per year.

TOXICITY STUDIES

Very little definitive dose-response information is available for evaluating the impact of low D.O. on fish. Correspondingly, it is exceedingly difficult to extrapolate these meager results to estimate the risk for man. However, it is essential to develop methods for doing such analyses for batch and variable flow and concentration systems if we ever hope to evaluate wet and dry-weather impacts.

WET-WEATHER STANDARDS

Efforts to specify a "design storm" have been fruitless because the relationship between rainfall and runoff, and receiving water impact is poorly understood. Results of the toxicity studies indicate that it is the continuous duration of a stated concentration level which is relevant if a concentration frequency curve approach is used. However, this approach is also of limited value unless we know not only the magnitude of the impact of controls, e.g., increase in D.O. days, but also its importance. There is no general way to value importance. It must be done on a case by case basis. For the same reasons, composite water quality indicators are of little value because they cannot be related directly to public health.

Given this state of uncertainty, a case by case analysis is recommended. A "problem" exists if there is direct evidence of impaired beneficial uses, e.g., periodic fish kills, beach closings. Controls can be applied selectively to ameliorate these problems.

PRODUCTION FUNCTIONS

The production function approach provides a simple but effective way to characterize the performance of controls. The aeration tank example indicates the importance of selecting production functions for control options using a consistent set of assumptions. While economists define a production function in terms of the maximum attainable output with a specified level of inputs, engineers modify this estimate to account for variability in performance. Actual designs include safety factors.

All of the production functions presented have diminishing marginal productivity throughout the range of performance. If marginal productivity is increasing initially, this implies that flow splitting can increase performance. These functions can be presented graphically or analytically. It is desirable to choose convenient functional forms, such as Cobb-Douglas, to expedite the subsequent economic analysis. Mathematical programming representations can be used for larger problems.

COST-EFFECTIVENESS ANALYSIS

The cost function should be generated as a guide to selecting a "reasonable" level of effectiveness. This is vividly illustrated by the nationwide cost assessment wherein about 75% of the BOD_5 can be removed for $25/acre ($62.00/ha). Costs to control beyond that level increase rapidly to about $125/acre ($309.00/ha) for about 84% control.

Unresolved problems remain in estimating unit costs because of widely varying assumptions about how common or overhead costs should be allocated. Marginal costing should not be used unless the model includes a method for assigning other costs. Game theory offers a good way to resolve this difficult problem by linking the economic and financial analysis.

FINANCIAL ANALYSIS

Existing methods are inadequate in that they do not always assign costs in a fair manner. Current Federal policies permit local agencies to cost share only the capital costs of a project thereby inducing local agencies to favor capital intensive projects.

No general theory can be developed to cover all conceivable cases of financial analysis. What is "fair" depends heavily on precedent and present perceptions. Nevertheless, game theory does provide an excellent theoretical and practical foundation for comparing alternatives. The minimum cost, remaining savings (MCRS) method is recommended but further development of these concepts is needed.

CHAPTER 3

DAMAGE ASSESSMENT

INTRODUCTION

Following a decade of an aggressive national water pollution control program, a significant portion of this problem has been eliminated. Most cities have installed secondary treatment plants which remove about 85-90% of the organic matter. Major industrial and agricultural waste sources have been controlled. Now attention has shifted to installing tertiary treatment plants to further control these known point sources. In addition, control of wet-weather pollution associated with combined sewer overflows is being implemented and control of urban and non-urban runoff is under serious consideration.

The national commitment to continued large expenditures for water pollution control is being reevaluated because it is being accused of causing inflation -another national problem of major concern (1,2). The cost of controlling the remaining water pollution is enormous with estimates ranging from 10 to 400 billions of dollars depending on the selected level of control (3,4,5).

It is axiomatic that diminishing marginal returns are associated with continued expenditures in any endeavor and water pollution control is no exception. This chapter reviews alternative ways to examine the question of how much water pollution control is necessary. An attempt is made to step back a bit from the usually accepted criterion for pollution control, i.e., to satisfy specified standards. This broader policy perspective is examined in the next section. Then the discussion moves from this general level to more specific and operational procedures for estimating the effectiveness of investments in water pollution control.

POLICY FRAMEWORK

An illustration of an overall framework within which water pollution policies are formulated is presented in Figure 3-1 (6). At the most general level is promotion of the general welfare of society. This broad social aim is accomplished through maintenance of security and enhancement of opportunity. Security is maintained by protecting environmental quality. Within this context, water quality management is a subgoal or objective. Social indicators of water quality include dissolved oxygen. Next, an operational description of specific policy actions to improve dissolved oxygen are presented. Much of the effort of technical people in the pollution control

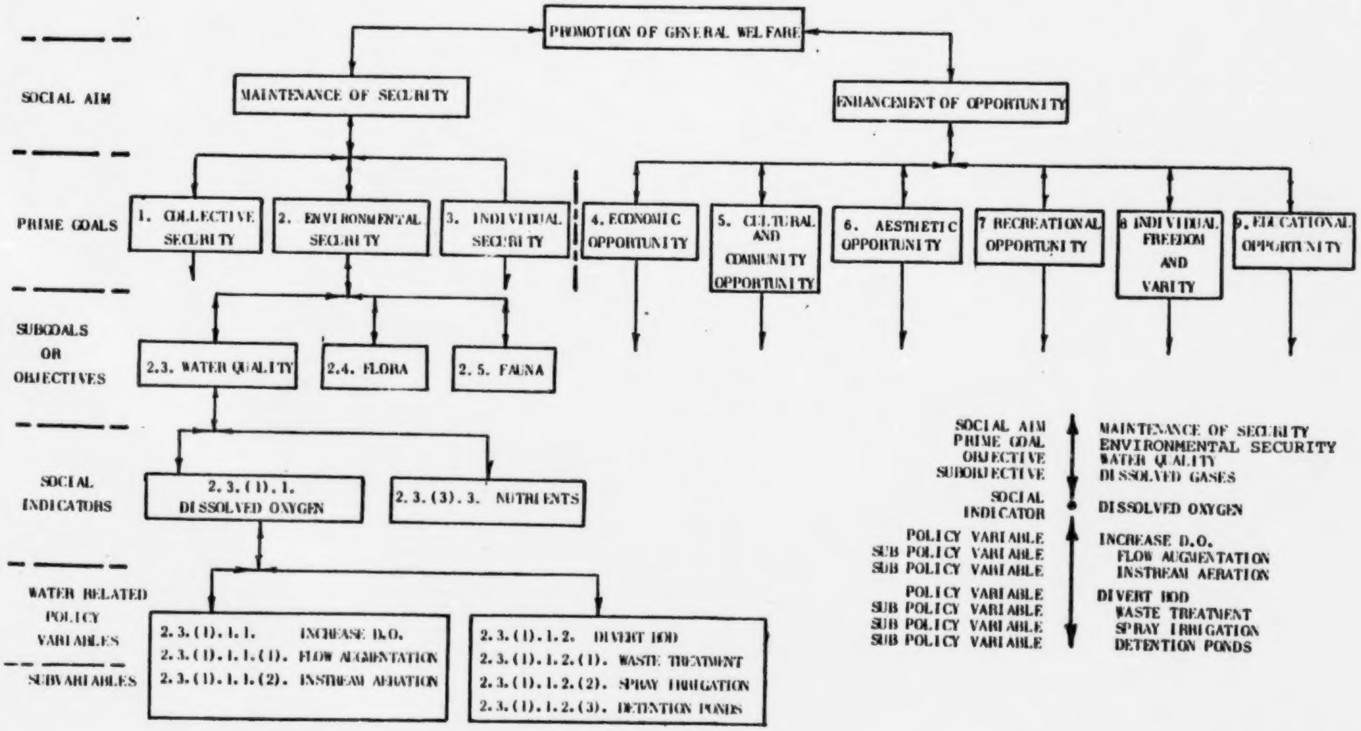
PROMOTION OF GENERAL WELFARE
SOCIAL AIM
MAINTENANCE OF SECURITY
ENHANCEMENT OF OPPORTUNITY
PRIME GOALS
1. COLLECTIVE SECURITY
2. ENVIRONMENTAL SECURITY
3. INDIVIDUAL SECURITY
4. ECONOMIC OPPORTUNITY
5. CULTURAL AND COMMUNITY OPPORTUNITY
6. AESTHETIC OPPORTUNITY
7 RECREATIONAL OPPORTUNITY
8 INDIVIDUAL FREEDOM AND VARITY
9. EDUCATIONAL OPPORTUNITY
SUBGOALS OR OBJECTIVES
2.3. WATER QUALITY
2.4. FLORA
2.5. FAUNA
SOCIAL INDICATORS
2.3.(1).1. DISSOLVED OXYGEN
2.3.(3).3. NUTRIENTS
WATER RELATED POLICY VARIABLES
SUBVARIABLES
2.3.(1).1.1. INCREASE D.O.
2.3.(1).1.1.(1). FLOW AUGMENTATION
2.3.(1).1.1.(2). INSTREAM AERATION
2.3.(1).1.2. DIVERT BOD
2.3.(1).1.2.(1). WASTE TREATMENT
2.3.(1).1.2.(2). SPRAY IRRIGATION
2.3.(1).1.2.(3). DETENTION PONDS
SOCIAL AIM
PRIME GOAL
OBJECTIVE
SUBOBJECTIVE
SOCIAL INDICATOR
POLICY VARIABLE
SUB POLICY VARIABLE
SUB POLICY VARIABLE
POLICY VARIABLE
SUB POLICY VARIABLE
SUB POLICY VARIABLE
SUB POLICY VARIABLE
MAINTENANCE OF SECURITY
ENVIRONMENTAL SECURITY
WATER QUALITY
DISSOLVED GASES
DISSOLVED OXYGEN
INCREASE D.O.
FLOW AUGMENTATION
INSTREAM AERATION
DIVERT BOD
WASTE TREATMENT
SPRAY IRRIGATION
DETENTION PONDS

Figure 3-1. Overall Framework for Water Pollution Policies (6).

field is directed at finding cost-effective solutions for given social indicators, e.g., dissolved oxygen shall equal or exceed 5 mg/l. Other goal hierarchies similar to Figure 3-1 could be used. The purpose of presenting Figure 3-1 is to emphasize that such a hierarchy exists and that control programs are not ends in themselves but rather ways, hopefully effective ones, of accomplishing social goals. The next section examines the environmental security framework within which water pollution control activities function.

ENVIRONMENTAL HEALTH

Attempts have been made to define "environmental quality" and "quality of life" as they relate to policy formulation. However, perusal of an anthology of selected readings from an EPA sponsored symposium on this subject indicates that this is, at best, a very nebulous subject (7). Thus, more direct measures of performance are needed.

The generally accepted broad goal of environmental quality management is to protect and enhance man's physical and mental health. The goal has an anthropocentric orientation, i.e., it is based on man's value system as contrasted to some natural value system (8). Other value criteria such as ecosystem stability have been proposed. However, Ehrenfeld points out that no consensus exists as to the optimal amount of diversity, or the nature of the loss function if the system is modified by man (8). Of course, man depends on the natural system for survival so its value is imputed in terms of its importance in protecting man's well being. No claim is made that this valuation system is flawless but it is essential to use a common norm so that investments in water pollution control can be compared to other forms of pollution control; or to other investments in environmental health such as public health clinics, rat eradication programs, etc. Within this context, alternative investment opportunities can be ranked in terms of their relative effectiveness in improving public health.

Eisenbud presents a current summary of health impacts associated with environmental pollution (9). Interestingly, he feels that too much money is being invested in air and water pollution control programs in New York and that these monies would be better spent on other public health controls. Specific procedures for environmental health impact assessment are described by Morris and Novak (10).

Starr addresses the related question of risk in engineering design in which people individually, e.g., making travel plans, and collectively, e.g., flood control works, assess the riskiness of various courses of action (11). A collective notion of risk is involved in establishing design criteria for a wide variety of public facilities. In all cases, an explicit or implicit assessment of risk is made.

Richard Wilson of Harvard U. describes the results of attempts to implement a public policy which eliminates the risk of cancer at any cost (12). As an example, he cites a proposed OSHA program which would cost about $300 x 10^6 for every life saved, about one fourth of the lives that would be lost implementing the proposed controls. Wilson's main point is that public policy formulations need to recognize that there is risk involved in all of man's activities. The public policy analysis seeks to determine an acceptable level. For example, according to Wilson, each of the following activities can be expected to reduce life expectancy by 8 minutes.

Smoking 1.4 cigarettes
Living two months with a cigarette smoker
One X-ray (in a good hospital)
Eating 100 charcoal broiled steaks
Eating 40 tablespoons of peanut butter
Drinking 1000 24-ounce (710 ml) soft drinks from recently banned plastic bottles
Drinking 30 12-ounce (355 ml) cans of diet soda containing saccharin
Spending three hours in a coal mine
Traveling 10 miles (16 km) by bicycle
Traveling 1000 miles (1610 km) by jet

Related examples have appeared in flood control wherein the expected number of construction workers killed building a flood control reservoir exceeds the expected loss of life from flooding (13). Quite recently, a tragic accident killed 54 workers constructing a cooling tower to control thermal pollution in West Virginia. Similar concerns have been voiced about the wisdom of controlling organics in drinking water (14).

At present, no explicit trade off analysis is performed in allocating resources among various environmental health activities. Rather, each one promulgates standards of performance which are felt to provide an "acceptable" level of control. The remaining sections explore the feasibility of quantifying pollutant damages.

WATER QUALITY BENEFITS

Two levels of benefit analysis may be performed for water quality management problems: 1) quantify the physical magnitude, e.g., 1000 visitor days, and 2) quantify the magnitude and importance, e.g., 1000 visitor days and $5 per visitor day. In pollution control studies, benefits are estimated in terms of reduction in damages as shown in Figure 3-2. Dual ordinates are included to indicate that damages may be measured in a physical unit such as number of fish killed or it may be expressed in terms of the monetary equivalent of these damages.

Damages are reduced by installing pollution control devices. If the pollution control program is properly managed, priority will be given to amelioriating the most serious damages. For example, the first million dollars would be spent protecting a community water supply rather than improving the aesthetics of a wilderness area stream. Thus, the damages curve has the characteristic shape shown in Figure 3-2. Reorienting Figure 3-2 yields a direct plot of benefits as a function of % pollution control as shown in Figure 3-3. This curve exhibits the typically observed diminishing marginal value, i.e., each additional % pollution control yields progressively smaller increases in benefits.

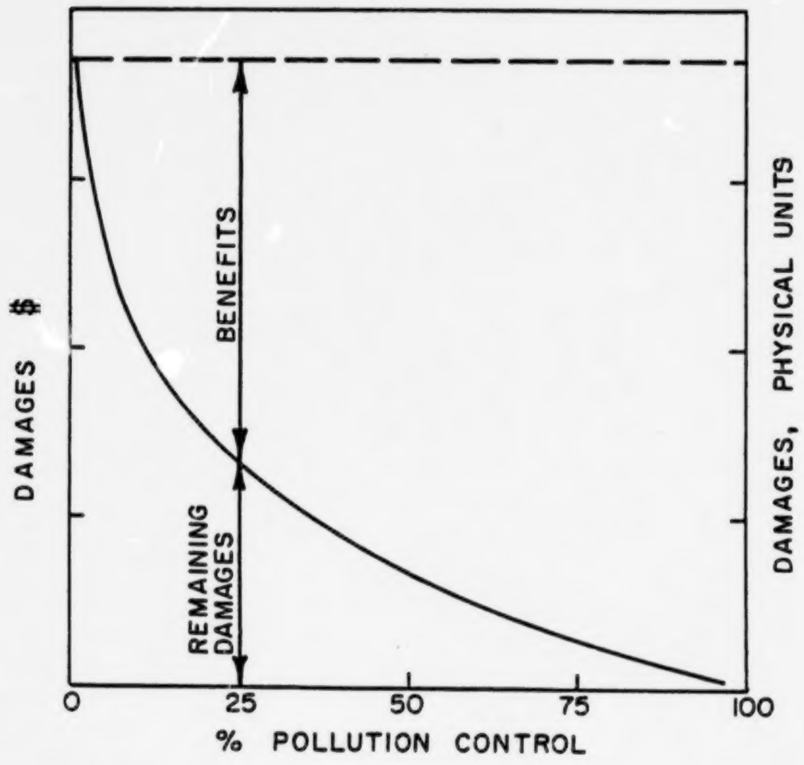

Figure 3-2. Benefits Measured as Reduction in Damages.

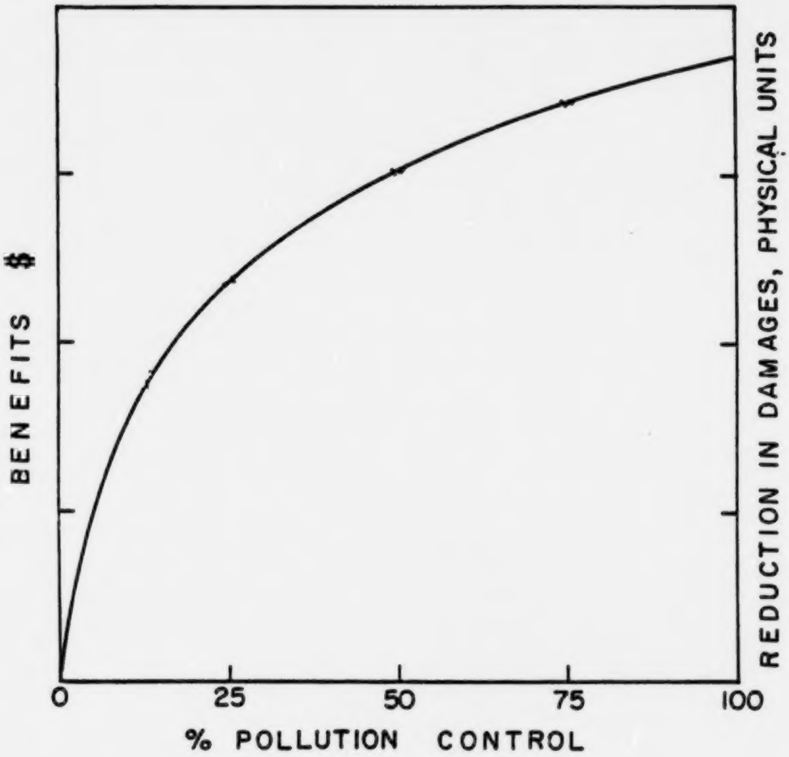

Figure 3-3. Benefits as a Function of % Pollution Control.

A review of the literature indicates that only crude comparisons can be made between levels of pollution control and expected benefits. (See Tihansky (15) for general information on this subject.) Thus, functions relating, say D.O. concentration, to visitor days, are generally unavailable as D.O. is only one of several water quality parameters affecting the public's perception of water quality. Thus, only a few discrete physical conditions of the receiving water may be discernible, e.g., dirty or clean.

The other level of benefit analysis wherein a valuation is included can become very complicated. At the simple end, a single unit valuation could be used, e.g., $10 per capita per year. On the other extreme, a detailed equilibrium analysis directed at determining the net change in value from implementing controls could be made. In light of the uncertainty in estimating the magnitude of benefits, a sophisticated valuation procedure seems unwarranted at this time. Rather, using a range of typical values will at least give the decision maker a rough idea of the relative value of pursuing alternative courses of action. For example, the National Commission on Water Quality (NCWQ) studies on control costs indicated that \$357 x 10^9 would be required to satisfy PL 92-500 (3). The approximate per capita annual cost equivalent is about $180 per year.

A significant part of the effort of the NCWQ dealt with quantifying benefits (3). The results of their studies, summarized in Table 3-1, provide an estimate of the benefits of satisfying the requirements of PL 92-500. Assuming a uniform distribution of benefits among the 200 x 10^6 citizens, the per capita benefits are $28.00 per year. Note that 70% of the total benefits are attributable to enhanced fishing in coastal areas. In addition to these quantifiable benefits, there are potential public health benefits, and the value of avoiding irreversible environmental damages.

Based on studies of recreational expenditures as related to water quality, Russell concludes that ". . . one may expect the per capita willingness to pay of the domestic sector to be on the order of $5 per year for a significant improvement in quality conditions, one permitting a wide range of recreational activities on a stream previously closed to most or all of these" (16).

A recently completed case study of the benefits from improving water quality in the South Platte River Basin in Colorado indicated average per capita benefits of $26.50 (17). This value is comprised of $11.50 for direct recreational benefits, $4.50 for "option" value; $6.25 from "existence" value; and $4.25 for "bequest" value. "Option" values are defined as the value of avoiding irreversible mineral and energy pollution. "Existence" value is defined as the value derived from knowing that this system is available for use. "Bequest" value refers to the estimated worth of protecting the system for posterity.

Converting the NCWQ 1985 estimate to 1980 values (at 8% discount rate) yields a 1980 per capita value of about $19.00/yr. Likewise, converting Russell's 1970 value to 1980 dollars yields an estimate of about $10.80 per capita per year. The South Platte study by Walsh et al. indicated per capita values ranging from $11.50 to $26.50 per year.

Benefit Category	1985 Value $\$ \times 10^6$/yr
1) Enhanced recreation marine fishing[a]	3300
2) Enhanced commercial marine fishing[b]	600
3) Enhanced boating[c]	850
4) Enhanced freshwater fishing[d]	370
5) Enhanced public swimming	
a) New activity[e]	90
b) Diverted activity[f]	250
6) Enhanced real estate values near related water bodies[g]	90
TOTAL	$5550

[a] Assuming additional sports fishing at a unit value of $13.90 per sports fishing day.

[b] About 8% due to shellfish bed openings. The balance is attributed to water quality induced productivity increases causing projected price reductions ranging from 10 percent (for lobster) to more than 50 percent (for clams).

[c] Assumes a unit value of $10 per boating day.

[d] Assumes a unit value of $8.30 per fishing day.

[e] Assumes new swimming activity valued at $7.50 per activity day.

[f] Assumes diverted swimming activity valued at $2.50 per activity day.

[g] Assumes aggregate increase in market value of residential property of $770 million capitalized at an annual rate of 12 percent.

Table 3-1. National Commission on Water Quality (NCWQ) Estimates of the Benefits of Meeting PL 92-500 Requirements (3).

Swimming pools can be viewed as an alternative to installing controls to render receiving waters swimmable. The annual costs of a swimming program would range from about $5 per capita for community pools to about $200 per capita for pools for every dwelling unit. It is absurd to view individual swimming pools as a "justifiable alternative" to swimming in the receiving water; however, it does serve to place an upper bound on this type of benefit.

The above estimates of the benefits from water quality improvement indicate a wide range in values. The relationship of water quality benefits to pollutant concentrations is unclear. The criteria used to open or close areas to swimming and/or fishing vary widely. Scientific evidence does not exist to permit examination of the impact of small changes in pollutant loads. Rather one can barely discern the difference between the major control levels: primary, secondary, and tertiary. On the positive side, the monetary estimates do get us in the ballpark. As an illustration, an annual per capita benefit of $10 might be used to compare to the cost of a proposed tertiary treatment plant which would open up a section of a receiving water to swimming and fishing. Correspondingly, if installation of such an expensive control will not affect the level of these activities, then perhaps the money would be better spent elsewhere. The next section evaluates the role of toxicity studies in providing a basis for assessing effectiveness.

TOXICITY STUDIES

As mentioned earlier, the magnitude of receiving water impacts is difficult to estimate. An even more difficult problem is to estimate the importance of these observed levels. A general epidemiological procedure involves determining the dose-response curve for the receptor under consideration, e.g., fish, man. An equivalent deleterious effect results from exposure to a high concentration for a shorter time, or a low concentration for a longer time. Unfortunately, these functions are very difficult to obtain. Without such a relationship, it is impossible to quantify damages from pollution. A review of the literature did not reveal sufficient information to quantify a general damage function. Thus, additional work was done on this subject using fish toxicity as the measure of damage (18).

Many pollutants can damage a fishing resource. Acids or bases can alter the pH to unacceptable levels. Excessive nutrients can cause a detrimental shift of the algal population. Thermal discharges can affect fish by thermal shock or long-term elevated temperatures. However, of the many pollution caused changes of water quality that can adversely affect fish, none is generally considered more important than reduction of dissolved oxygen (D.O.) (19). A review of yearly pamphlets entitled "Pollution Caused Fish Kills," which were published from 1964 through 1969 by the Public Health Service, indicates that low dissolved oxygen concentrations were the main cause of fish kills during this period (20).

State water quality standards generally prohibit the D.O. from falling below 5 mg/l during the once-in-ten-year seven-day low flow (7Q10). This implies an infinitely sloped damage function at 5 mg/l. Total damages do not, of course, occur at 5 mg/l but this was established as the standard because concentrations of 5 mg/l and higher are known to support a healthy diverse fish population (21).

Certain substances, such as cyanides, are toxic to aquatic life including fish. An illustrative toxicity reaction is shown in Figure 3-4 (22). As the graph indicates, in relating a toxicant and a physiological response, two variables are important: (1) the concentration of the agent, and (2) the continuous duration of the exposure to that concentration. Wilber presents several toxicological examples to illustrate that the general shape of the curve is as shown in Figure 3-4. The function is asymptotic to both axes. Also Chen and Selleck's data show the same type of relationship (23).

Neither Wilber (22) nor Chen and Selleck (23) discuss the toxicity of low dissolved oxygen concentrations. However, in addition to toxic compounds such as cyanides, low D.O. concentrations are also toxic to aquatic life. D.O. toxicity is distinguished from other compounds in that low concentrations place stress on the organisms rather than high concentrations. Figure 3-5 illustrates the effect of low D.O.'s for various duration times on the larvae of the aquatic insect Ephemera simulans (24). The physiological responses plotted in Figure 3-5 are the killing of a certain portion of the population. For example, LC_{20} refers to the lethal concentration to kill 20 percent of the population and is a function of concentration and duration of exposure. Below the LC_{20}, a line of similar shape, LC_0, must exist and below this a "no stress" line must exist. No data are available for either one of these lines, but hypothetical LC_0 and "no stress" lines have been included in Figure 3-5 for illustrative purposes.

The intepretation of Figure 3-5 is clear for a single event. As an example, a constant concentration of 4 mg/l for 8 consecutive days will kill about 20 per cent of the population of this aquatic insect. However, the sublethal effect of this event on the remaining 80 per cent of the population is unclear and the effects of a second similar event are unknown. If the sublethal effects of the first event were negligible, a second similar event would not cause a kill because no susceptible organisms would be present. If the sublethal effects were of sufficient magnitude to redistribute the remaining population to the original frequency distribution, the second similar event would kill 20 percent of the remaining 80 percent, i.e., 16 percent. There is no evidence to indicate that either one of these assumptions concerning sublethal effects is correct (22).

Lacking such information, the two extremes can be examined. Case 1 assumes sublethal effects are negligible. Thus, the proportion of the population killed would depend only on the maximum kill of all the events. Case 2 assumes that sublethal effects are sufficient to redistribute the remaining population to the original distribution after each kill. This case would represent the upper bound on the overall kill. This argument can be formalized as follows:

Let N_o = initial population,

N_i = population after $i^{\underline{th}}$ killing event,

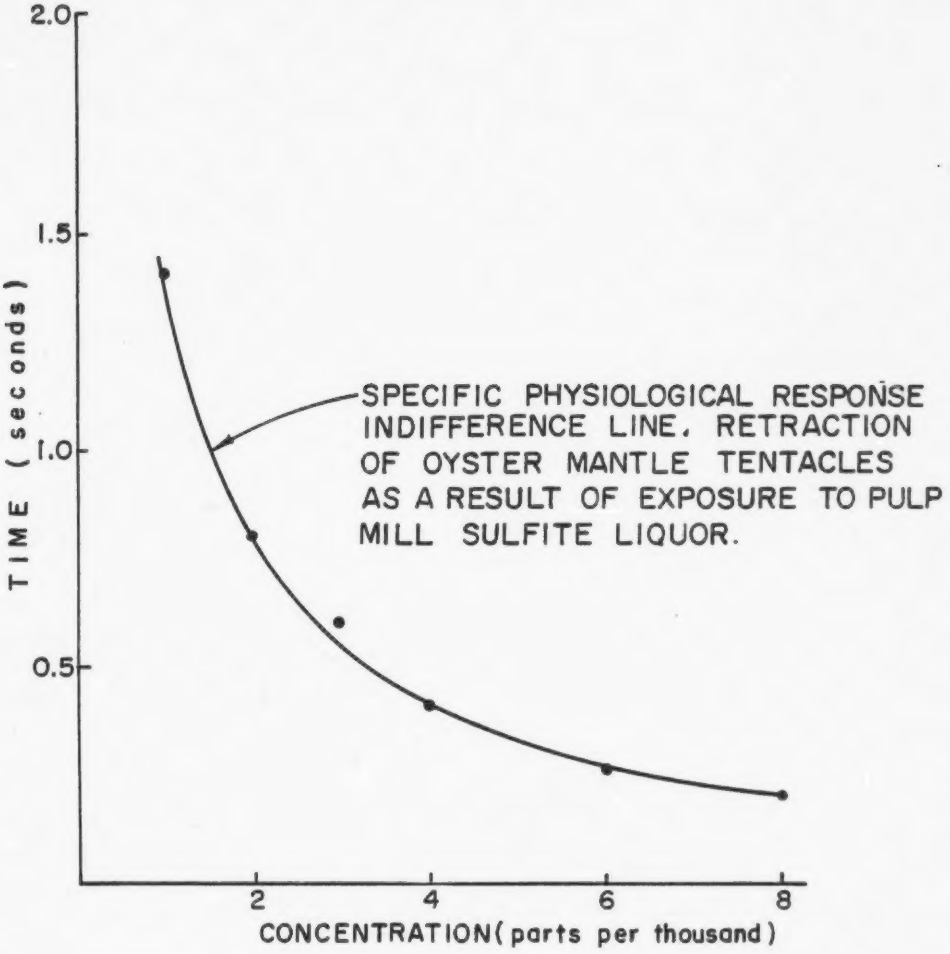

Figure 3-4. Generalized Toxicity Relationship Between Concentration and Duration to Accomplish a Physiological Response. (22)

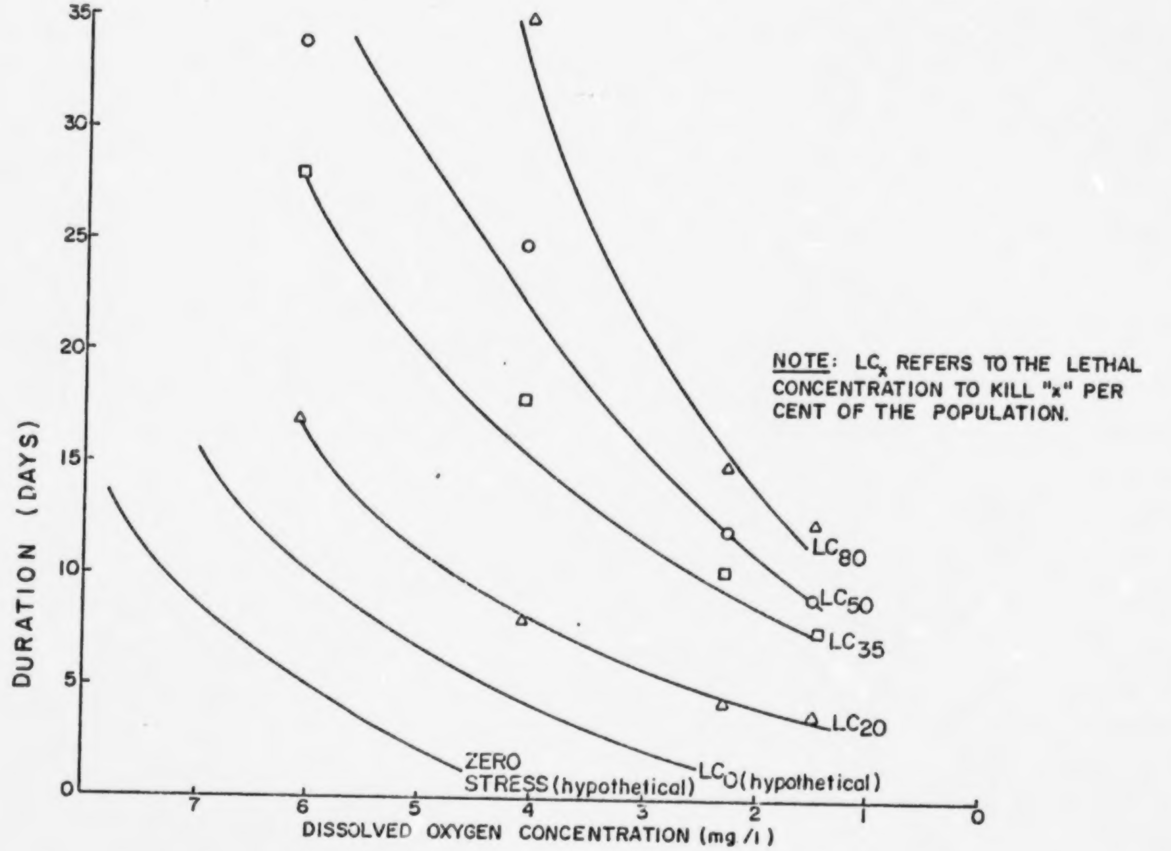

Figure 3-5. Dissolved Oxygen Duration Toxicity Relationship for Ephemera Simulans, an Aquatic Insect (Larvae) (24).

N_n = final population, and

k_i = proportion of population killed by $i^{\underline{th}}$ event.

Then, for Case 1

$$N_n = (1 - k_{i\ max})N_o \qquad (3\text{-}1)$$

where $(k_i)_{max}$ denotes the maximum kill of any event. For Case 2, the surviving population after event 1, N_1, is

$$N_1 = (1 - k_1)\ (N_o) \qquad (3\text{-}2)$$

After event 2, the surviving population is

$$N_2 = (1 - k_2)\ (N_1) \qquad (3\text{-}3)$$

The final surviving population, N_n, is

$$N_n = (1 - k_n)\ (N_{n-1}) \qquad (3\text{-}4)$$

Substituting equation 3-2 into 3-3 yields

$$N_2 = (1 - k_1)\ (1 - k_2)\ N_o \qquad (3\text{-}5)$$

Continuing in this manner, the final population, N_n, is

$$N_n = \prod_{i=1}^{n} (1 - k_i)\ N_o \qquad (3\text{-}6)$$

As mentioned above, it is not known whether Case 1 (Eqn. 3-1) or Case 2 (Eqn. 3-6) is correct. As a compromise, the two results are averaged. Then, the expected proportion of the population remaining after n killing events is

$$\frac{N_n}{N_o} = \frac{(1 - k_i)_{max} + \prod_{i=1}^{n} (1 - k_i)}{2} \qquad (3\text{-}7)$$

EXAMPLE

During a hypothetical year, four "low dissolved oxygen events" occurred in stream A which had an initial population of 1000 aquatic insects. The per cent reduction in the population is computed below.

(successive daily dissolved oxygen concentrations)
Event 1 7.0, 5.8, 4.3, 3.6, 2.5, 2.6, 2.8, 3.0, 3.2, 3.8, 4.4, 5.0, 5.8, 7.3

Event 2 7.0, 4.2, 1.8, 1.6, 1.4, 1.3, 1.3, 1.2, 1.4, 1.6, 1.8, 2.9, 2.7, 7.0

Event 3 7.6, 5.0, 3.4, 3.3, 3.2, 3.1, 3.3, 3.4, 3.4, 3.4, 3.1, 3.0, 3.4, 3.4, 3.6, 3.8, 4.5, 5.0, 7.0

Event 4 7.6, 5.7, 4.1, 1.8, 1.2, 1.3, 1.2, 1.2, 1.3, 3.2, 4.0, 4.2, 4.8, 5.9, 7.0

Find the proportion of the insects remaining for cases 1 and 2 and the average of the two cases.

Solution

1. Rank the data for each event in terms of number of consecutive days that the D.O. was less than or equal to indicated value. The results for event 1 are:

No. of consecutive days D.O. ≦ indicated value	D.O. concentration, mg/1
1	2.5
2	2.6
3	2.8
4	3.0
5	3.2
6	3.6
7	3.8
8	4.3
9	4.4
10	5.0
12	5.8
13	7.0
14	7.3

2. Plot the above data as shown in Figure 3-6 and find the maximum proportion of the species killed. The results indicate the following values for k_i

$$k = [\ .23,\ .34,\ .33,\ .24\ ]$$

3. Find the proportion of the insects remaining for case 1. From equation 3-1

$$\frac{N_n}{N_o} = (1 - .34) = .66$$

For case 2, from equation 3-6,

$$\frac{N_n}{N_o} = (1 - .23)(1 - .34)(1 - .33)(1 - .24) = .26$$

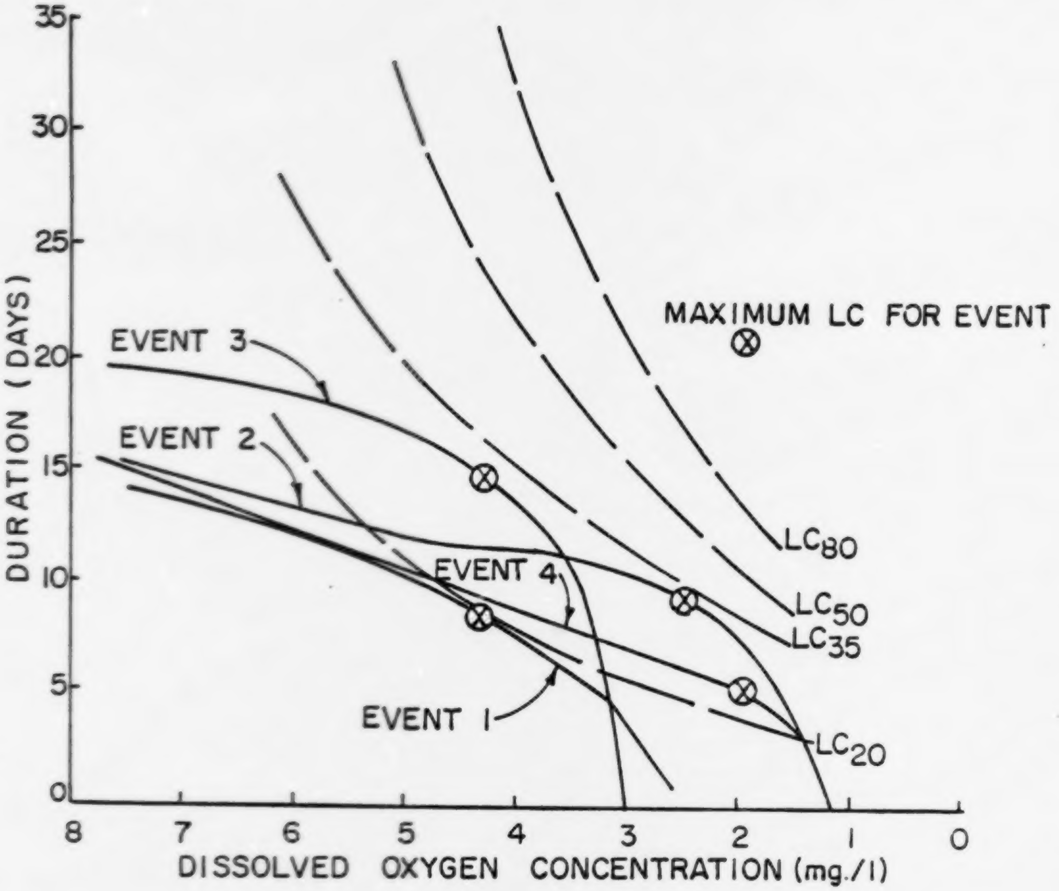

Figure 3-6. Finding the Maximum Lethal Concentration for an Event.

The average of the two cases is

$$\frac{N_n}{N_o}_{avg} = \frac{.66 + .26}{2} = .46$$

or slightly less than one-half of the insects surviving.

RECEIVING WATER STANDARDS

For river systems, the commonly used criteria is that the concentration of dissolved oxygen shall be at least 4 or 5 mg/l during the once in ten year low flow of seven day duration. This standard evolved based on toxicity studies on fish, aesthetic considerations, etc. (see 25-28 for a comprehensive survey of criteria). As an example, for dissolved oxygen, the influential publication Water Quality Criteria (25) suggests that ". . . the minimum permissible oxygen concentration for a well-rounded warm-water fish population be as follows: The dissolved-oxygen content of warm-water fish habitats shall be not less than 5 mg/l during at least 16 hours of any 24 hour period. It may be less than 5 mg/l for a period not to exceed 8 hours within any 24 hour period, but at no time shall the the oxygen content be less than 3 mg/l."

For toxic substances, the FWPCA's Water Quality Criteria (27) recommends that the allowable concentration of a toxicant be based on the concentration which was shown to kill 50% of the species under study, i.e., the TL_m. The recommendation is as follows:

> "Concentration of materials that are nonpersistent (that is, have a half-life of less than 96 hours) or have noncumulative effects after mixing with the receiving water should not exceed 1/10 of the 96 hour TL_m value at any time or place. The 24-hour average of the concentration of these materials should not exceed 1/20 of the TL_m value after mixing. For other toxicants, the concentration should not exceed 1/20 and 1/100 of the TL_m value under the conditions described above."

At present, the primary water quality parameters for which specific regulations exist are dissolved oxygen, pH, turbidity, suspended solids, and coliforms. A review of the literature indicates that no clearly defined dose-response relationships exist for dissolved oxygen, suspended solids, pH, or turbidity. Coliforms are used as public health indicators for swimming and recreational waters. However, no clear relationships exist between pollutant levels and public health hazards from swimming. Even where gross pollution exists, the published evidence is inconclusive (25).

Thus, existing receiving water quality standards are not based on direct measures of man's health but rather on fish toxicity, aesthetics, etc. None of these measures are easily transformed into the risk for man.

WET-WEATHER STANDARDS

Attempts to develop water quality standards for wet-weather discharges further complicate the already difficult dry-weather water quality standards problem. Efforts to specify a "design storm" have been fruitless because the relationship between rainfall and runoff, and receiving water impact is poorly understood. Exploratory work has been done in evaluating the continuous response of a receiving water to dry- and wet-weather pollution. Models are available for performing such analyses, e.g., Hydrocomp Simulation Program and EPA SWMM. Sample results for a Level III SWMM application to Des Moines, Iowa are presented in Figure 3-7 (29,30). These frequency curves tell the percent of time during the period of interest that the D.O. equalled or exceeded the indicated value. The "benefit" from installing a control can be estimated as the area between the frequency curves with and without the indicated control. For this application, the units are in D.O.-days - a measure of the magnitude of the change but not its importance. There is no general way to value the importance. It must be done on a case by case basis. Nevertheless, this approach is useful if the policy question is presented in this context.

The results of the toxicity studies do provide some guidance in the move towards establishing performance standards for wet-weather pollution. These tests indicate that it is the continuous duration which is relevant, not simply the relative frequency as shown in Figure 3-7. Thus, these frequency curves should be modified accordingly.

OTHER MEASURES OF EFFECTIVENESS

In a general way, water pollution is a composite of numerous wastes in a suspended, colloidal, or dissolved state in the water. Because such a large number of pollutants exist, only a small subset of these pollutants are selected as "indicators." The three most commonly used indicators are:

Indicator
Biochemical Oxygen Demand (BOD)
Suspended Solids
Fecal Coliforms

In addition, nutrients, heavy metals, pesticides, and other pollutants may be of concern. A receiving water quality standard which limits the concentration of each pollutant may be specified. This approach can be criticized because it ignores the other pollutants.

As an alternative, some overall measure of "effectiveness" might be devised which provides an aggregate indicator of pollution control. One approach is to devise an environmental quality indicator which provides a measure of the overall impact of the pollutants. Most of these indicators are additive models in which environmental quality (EQ) is a weighted function of selected water quality parameters (see Dunette (31) or Ott (32) for a summary of the literature on the subject). While water quality indicators are growing in popularity, the method has some serious theoretical and practical

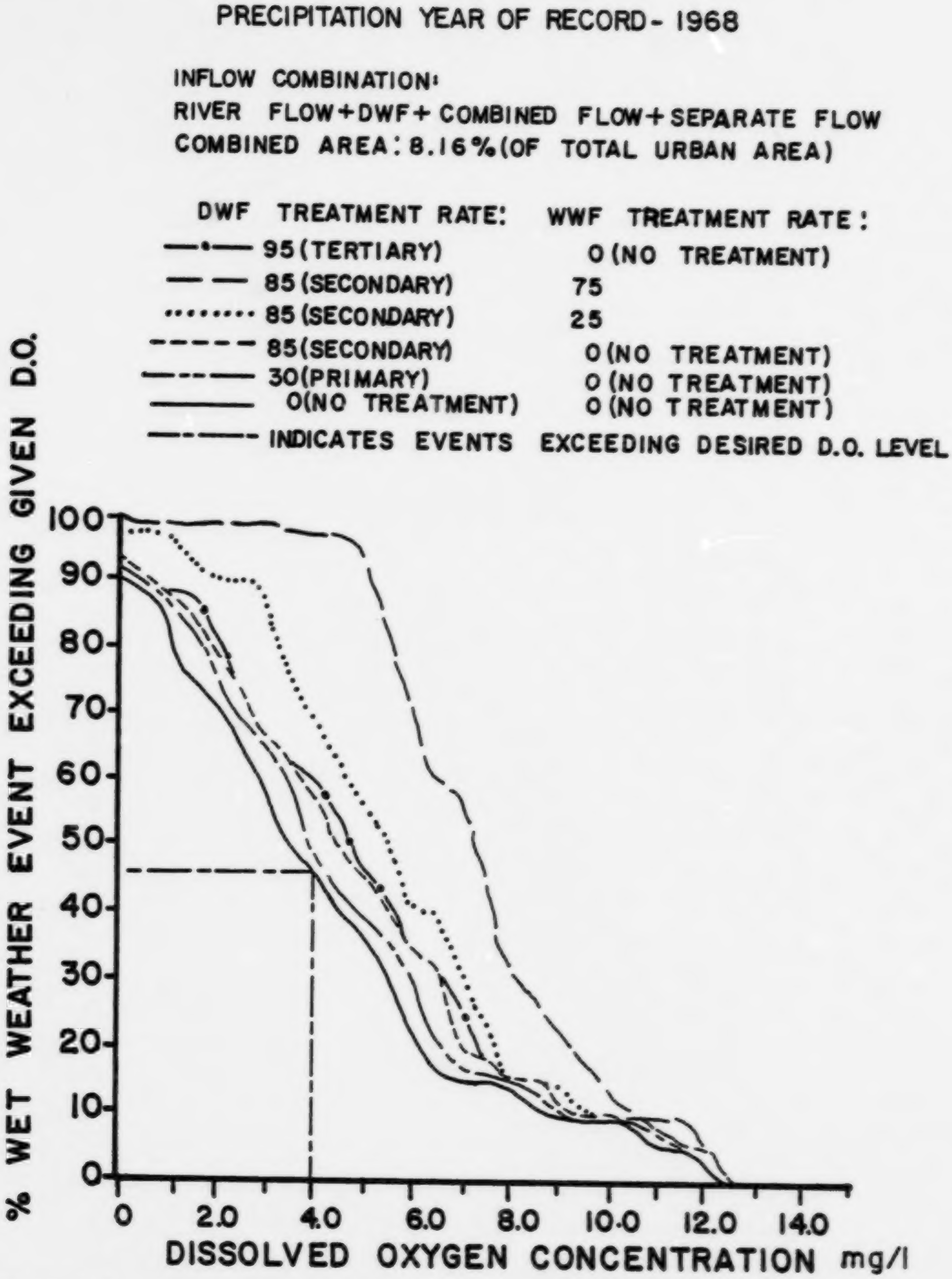

Figure3-7. Minimum D.O. Frequency Curves for Various Treatment Alternatives. (29)

limitations. For the purpose of this discussion, it is of little value unless it can be related directly to public health.

RECOMMENDATIONS

The previous sections have analyzed more general ways of evaluating receiving water impacts. It is evident that damage assessment is a difficult task because the most obvious pollution problems have been solved. The residuals discharges confronting us today have much more subtle impacts. Furthermore no agreed upon definition exists of what constitutes a problem. We have moved from simple single parameter measures of water quality to sophisticated ecosystem response analysis. However, these more refined approaches often further confuse, rather than elucidate, the underlying issues of whether a "problem" really exists.

Indeed, it appears that a sensible course of action is to examine these problems on a case by case basis. A "problem" exists if there is direct evidence of impaired beneficial uses, e.g., periodic fish kills, beach closings. Controls can be applied selectively to solve these "problems." As the number of such success stories grows, it should become possible to promulgate more general guidelines. Concurrently, research can continue on explaining the cause-effect relationships. This approach can be criticized in that it would give priority to areas where a problem is known to exist and perhaps miss another area with a more serious water quality problem which was not controlled because the affected public was not aware of the seriousness of the problem. But that leads us back to the discussion of overall social goals wherein the problem perception is part of the overall analysis. This attitude recognizes the desires of some areas to maintain a very high level of water quality, e.g., protecting the salmon in the Pacific Northwest, while other areas perceive that their river is "clean enough."

Two excellent examples of this case by case approach as applied to New York (33) and Seattle (34) have been presented. These examples are illustrative of a half-way house in the move towards direct quantification of benefits. In both case studies, control effectiveness is related to impaired beneficial uses, i.e., closed beaches. However, in both cases, selective abatement was recommended only at those sources which directly affected beneficial uses. This approach provides decision makers with much more valuable information than simply stating that standards are or are not being met.

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CHAPTER 4

PRODUCTION FUNCTIONS FOR WATER POLLUTION CONTROL

INTRODUCTION

Aggressive enforcement during the past ten years has resulted in control of much of the water pollution from larger point sources such as cities and industries. The incremental costs of further controlling these sources is quite high. Concurrently, there is growing recognition that it may be more cost-effective to control wet-weather pollution sources from urban, industrial, agricultural, silvicultural, and other activities. From an economic point of view, the objective is to find that combination of control options which satisfies the specified environmental standard in the least costly manner. The controls to be considered include not only the traditional treatment plant options but a wide variety of so-called Best Management Practices (BMP's) including street sweeping, sewer flushing, erosion control devices, detention facilities, on-farm management practices, and so on.

A simple desktop method for evaluating such a wide variety of alternatives is available (1). A vital component in that procedure was the characterization of the performance of the control units in a simple and consistent manner. The selected characterization is called a production function, the accepted terminology in the field of economics. A production function is the summary representation of the *maximum* output which can be attained from using a given amount of inputs. It is the locus of *technically efficient* combinations of inputs and outputs. As such, it represents the total description of the physical, chemical, and biological processes. The emphasis on using the maximum output is intended to convey that the engineers have indeed found the best way to blend the inputs.

The next section provides some of the history of production functions and defines the terminology to be used. Then, a case study of the evolution of the production function for an aeration tank is presented to illustrate some of the practical problems in obtaining such functions. Next, a series of production functions are presented for a relatively wide variety of controls. Included in this presentation is the observation that the production functions of concern rarely have increasing marginal productivity as is commonly assumed in economics.

HISTORY OF PRODUCTION FUNCTIONS

Chenery (2), Heady and Dillon (3), and Walters (4) provide good summaries of early work on production functions. Pioneering work was done by

Cobb and Douglas (5) in their studies of the American economy. They developed the relationship between manufacturing output, P, and two inputs, labor, L, and capital, C, shown in equation 4-1.

$$P = 1.01\, L^{.75}\, C^{.25} \qquad (4\text{-}1)$$

One purpose of their study was to estimate the proportion of output attributable to labor and capital.

Tolley and his co-workers (6), in an early study of agricultural production functions, related the daily gain in weight for a cow to its consumption of corn and hay as shown in Table 4-1.

Corn, lb/day	Hay, lb/day			
	8	12	16	20
10	1.61	1.81	1.98	2.13
15	1.96	2.16	2.33	2.48
20	2.27	2.47	2.64	2.79
25	2.41	2.61	2.78	2.79

Table 4-1. Daily Gain in Weight for a Cow (lb/day) as a Function of Corn and Hay Consumption.

Chenery (2) presents a production function derived for heat transmission, y, as a function of heat input, x_1, and the amount of insulation, x_2, i.e.,

$$y = x_1 - \frac{280}{0.0138 + 0.000595x_2} \qquad (4\text{-}2)$$

The data for these three production functions were obtained in different ways. Equation 4-1 was based on time series data for American industries over the period from 1899-1922. The agricultural studies of Tolley et al. were based on farm research. Lastly, Chenery's production function was derived based on the physics of heat loss. Thus, there are numerous ways to obtain data for a particular system. An important distinction needs to be made between functions derived with and without knowledge of the underlying causative mechanisms. Without such knowledge, a completely statistical approach is used which may provide an excellent fit to the data but be totally meaningless from a physical, chemical, and/or biological viewpoint. (See Wallis (7) for an example of the problem of deriving the "correct" functional relationship for the weight of a hollow cylinder by statistical methods only).

Mathematically, a production function can be written as

$$g(\bar{X},\bar{Y}) = 0 \qquad (4\text{-}3)$$

where $\bar{X}$ = vector of inputs $(x_1, \ldots, x_i, \ldots, x_n)$, and

$\bar{Y}$ = vector of outputs $(y_1, \ldots, y_j, \ldots, y_m)$.

For the simple case of one input and one output, the production function is usually depicted as shown in Figure 4-1. Given the function shown in Figure 4-1, the portion to the right of x_2 can be eliminated as being technically inefficient. Any point, say B, to the right of x_2 is dominated by a corresponding point A to the left of x_2 which provides the same output, y_1, using less input. Between x_1 and x_2, the "law" of diminishing marginal productivity holds. However, between 0 and x_1, marginal productivity increases. The usual argument is that the rational firm, if it decides to produce at all, will move at least to x_1. Thus, the only area of concern is in the range of $x_1 \leqq x \leqq x_2$. From the viewpoint of the subsequent economic optimization, it is better if the production function bounds a convex set. A convex set is one in which a line segment joining any two points in the set is contained wholly within the set. Referring to Figure 4-1, the line joining the origin and point C lies outside the feasible region. Later, a procedure will be presented for converting production functions such as shown in Figure 4-1 to convex sets.

Mathematically, properties of production functions can be described as follows:

a) Marginal Productivity, MP_{ij}

$MP_{ij} = \partial y_j / \partial x_i$ = increase in output y_j per unit increase in input x_i.

b) Average Productivity, AP_{ij}

$AP_{ij} = y_j / x_i$ = total production of output y_j divided by the total amount of input x_i.

c) Marginal Rate of Technical Substitution, $MRTS_{ki}$

$MRTS_{ki} = \dfrac{\partial x_k}{\partial x_i}$ = number of units by which input x_k can be reduced when use of input x_i is expanded by one unit so as to maintain a constant output.

Marginal and average productivity curves are included on Figure 4-1. The interested reader is referred to Ferguson (8), Henderson and Quandt (9), or similar texts for additional information on production functions.

The primary contribution of economists is their development of the general properties of production functions, the type of functional forms which can be manipulated easily, and how to interpret such functions. For example, the Cobb-Douglas type of production function, e.g., Equation 4-1, is very popular. Engineers attempting to find optimal solutions to process control problems often derive production functions which are unwieldy to manipulate. Awareness of the theory of production functions could simplify

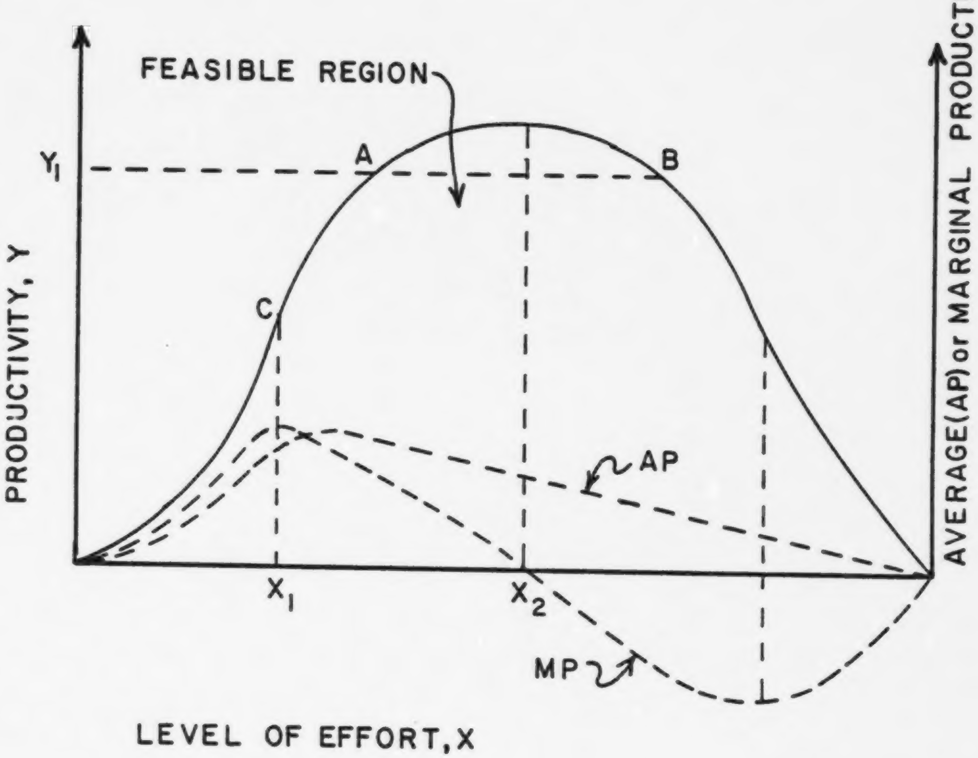

Figure 4-1. Generalized Production Function for One Input, One Output.

their analysis considerably. The next section describes various production functions for an aeration tank, a major process in activated sludge sewage treatment plants. This discussion illustrates some of the practical problems of developing meaningful production functions.

PRODUCTION FUNCTION FOR AERATION TANKS

Gagnon et al. (10) review and evaluate the parameters which have been and are being used for designing the aeration tank component of the activated sludge process. The earliest selected parameter was detention time. For example, Metcalf and Eddy (11) report the results of studies in Birmingham shown in Table 4-2.

	BOD Removal	
Aeration Time-hr.	Total %	Incremental, %/hr.
1	56	56
2	74	18
3	83	9
4	88	5
5	91	3
6	92	1

Table 4-2. Early Estimate of Aeration Tank Performance (11).

There is diminishing marginal productivity as seen in column 3, i.e., ever decreasing additional % control with each additional hour of residence time. With regard to economic analysis, costs are usually presented in terms of aeration tank volume rather than detention time. Fortunately, this is a minor problem since the relationship of the tank volume to detention time is known, i.e,

$$t = V/S \tag{4-4}$$

where t = detention time, hr., (sec)

S = tank volume, ft^3, and (m^3)

Q = flow rate, $ft^3/hr.$ (m^3/sec)

Based on early production functions such as Table 4-2, engineers began to establish rough design guidelines, i.e., the recommended aeration time is 4-10 hours for domestic waste treatment (10). This rule of thumb recognized that marginal productivities were still relatively high up to four hours so it is well to aerate for at least that long. On the other hand, continued aeration beyond 10 hours appeared to result in little, if any, additional removal.

Subsequent studies indicated that the organic loading per unit time (lb. BOD/day/1000 cu ft) provided a better single measure of aeration tank performance. Recommendations presented by Greeley (12) are presented in Table 4-3.

Unfortunately, the selected performance parameter is beginning to cause problems. First, removals increase as loading rate decreases resulting in an atypical type of production function form. This problem is easily resolved by using the inverse of the loading function, i.e., 1000 ft^3 per lb BOD/day.

Aeration Tank Loading lb BOD/day/1000 cu ft	% BOD Removal
240	40
200	50
160	60
120	70
80	80
40	90

Table 4-3. Aeration Tank Loading and % BOD Removal (12).

Second, it is much more difficult to find the unit costs as a function of aeration tank loadings.

The production function representation continued to grow in complexity as several modifications of the activitated sludge process were developed, e.g., step aeration, contact stabilization. Work continued on developing production functions based on statistical analysis of plant performance and/or laboratory and pilot plant process studies. Following their review of these studies, Gagnon et al. (10) present the results of a case study of the Jones Island wastewater treatment facility in Milwaukee. Using pounds of BOD per day per pound of mixed liquor volatile suspended solids as the performance parameter, they obtain the production function shown in Figure 4-2. The fuzzy set indicates the approximate variability in performance of the unit. Thus, even after decades of research and operating experience, significant variability remains. Accordingly, it is unwise to be too literal in interpreting production functions. While they are defined to denote the maximum output (BOD removal in this case) for a given input, this is not actually done. The engineering designer usually sizes the units with a built-in safety factor. Accordingly, a conservative estimate of performance is used, e.g., the lower boundary of Figure 4-2. Note also on Figure 4-2 that the relevant range over which performance is measured is quite narrow, i.e., $.80 \leqq y \leqq .98$, reflecting the general consensus that this is the proper operating range. However, with current interest in dry and wet-weather treatment we would like to consider operating this control over a much wider range of performance. Unfortunately, little data are available on the performance of these units when inflow and outflow rates vary considerably. For this unit in particular there is even a risk of a "shock load" killing the essential organisms and causing a shutdown.

The purpose of presenting this discussion is to indicate that a considerable amount of engineering judgement is involved in selecting not only this production function but also to ensure that the other production functions are developed under an internally consistent set of assumptions regarding

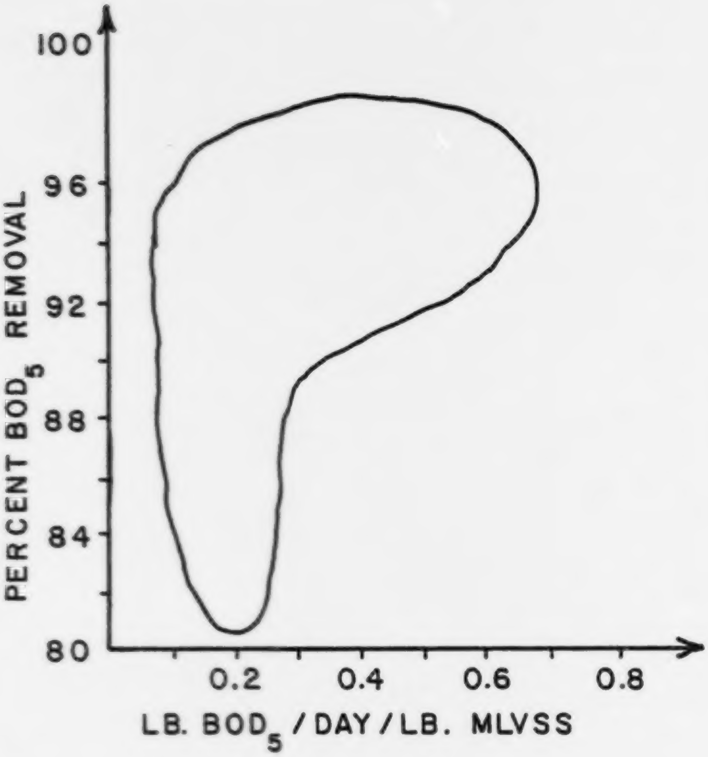

Figure 4-2. Production Function for Aeration Tank,

Milwaukee, Wisconsin (10).

safety factors, reliability, etc. The reader should also be cautioned to check carefully the quality of the data base used to construct this function. The next sections present several production functions for water pollution control studies.

PRODUCTION FUNCTIONS FOR WATER POLLUTION CONTROL SYSTEMS

Simple Wastewater Treatment Systems

The Areawide Assessment Procedures Manual (13) summarizes the pollutant removal characteristics of twelve different treatment systems as they impact on six different water quality constituents (see Table 4-4). The production function in this case is quite simple. Let $\bar{Y}$ = vector of pollutant removal outputs = $(y_1, \ldots, y_j, \ldots, y_6)$

where y_1 = proportion BOD_5 removed,

$\vdots$

y_j = proportion of pollutant j removed,

$\vdots$

y_6 = proportion UOD removed,

and $\bar{X}$ = vector of pollutant control inputs = $(x_1, \ldots, x_i, \ldots, x_{12})$

where x_1 = primary treatment; $x_1 = 0,1$

$\vdots$

x_i = control i; $x_i = 0,1$

$\vdots$

x_{12} = complete tertiary treatment; $x_{12} = 0,1$

Note that x_i only takes on values of 0 or 1 while y_j is continuous.

For example, the BOD_5 (j=1) removal if control 1 is used is

$$y_1 = \frac{(210 - 130)}{210} \quad x_1 = 0.38x_1$$

Table 4-4 is very important to environmental engineers since it presents, in a very compact form, the total characterization of the technical efficiency of the twelve most widely used treatment systems. It is the result of years of research studies and observations of full scale plant performance. Knowing these production functions, the associated costs of the system and the required performance standard, the economic optimization is quite straightforward.

Table 4-4. Wastewater Treatment System Production Function*

Values of a in $y_j = a_{ij} x_i$ $(x_i = 0,1)$

System Number i =	1 BOD_5	2 TSS	3 COD	4 Total P	5 NH_3-N	6 UOD[1]	Description of System
-	1.00	1.00	1.00	1.00	1.00	1.00	Raw Waste Characteristics
1	.38	.57	.38	.18	0	.30	Primary
2	.52	.78	.54	.82	0	.41	Primary and Metal Salt Addition ($FeCl_3$)
3	.79	.74	.78	.27	.10	.63	Primary and Trickling Filter
4	.88	.87	.88	.82	.10	.70	Primary and Trickling Filter with Metal Salt ($FeCl_3$)
5	.90	.91	.89	.36	.15	.74	Primary and Activated Sludge
6	.93	.93	.91	.82	.15	.75	Primary, Activated Sludge, Metal Salt (Alum)
7	.95	.91	.91	.27	.90	.94	Primary, Activated Sludge/Nitrification
8	.95	.91	.89	.27	.95	.95	Primary, Activated Sludge, Nitrification Denitrification
9	.95	.96	.89	.91	.90	.94	Primary Metal Salt Addition (Alum), Activated Sludge/Nitrification, Filtration
10	.98	.98	.94	.91	0	.76	Preliminary, Two-Stage Lime, Filtration, Carbon Adsorption
11	.98	.98	.92	.95	.95	.97	Primary Metal Salt Addition (Alum), Activated Sludge, Nitrification, Denitrification, Polymer, Filtration
12	.99	.98	.98	.95	.95	.97	Primary Metal Salt Addition (Alum), Activated Sludge, Nitrification, Denitrification, Polymer, Filtration, Carbon Adsorption

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Note: Treatment systems include disinfection, sludge handling, miscellaneous structures, and support personnel.

[1] UOD = Ultimate Oxygen Demand = (1.5 x BOD_5) + (4.57 x NH_3-N).

*Source: EPA, Areawide Assessment Procedures Manual. App. H., EPA-600/9-76-014, Cincinnati, OH, 1976.

In this case, the analysis was straightforward because the production functions relating resource inputs and pollution control as the output were very simple. However, the analysis required at present includes not only alternative treatment plant configurations but also equalizing storage-treatment systems for wastewater control and wet-weather control, and best management practices such as street sweeping, sewer repair and/or flushing, etc. In addition, opportunities exist for multi-purpose schemes, e.g., storage for drainage control and pollution control. The next example deals with the problem of equalizing storage.

Equalizing Storage in Sewage Treatment

The value of the production function approach can be demonstrated by examining the common problem of evaluating the desirability of flow equalization. This example is taken from Clark and Viessman (14). The hourly influent sewage flow on the maximum day is shown in Figure 4-3. Typically, two alternatives are examined:

a) Construct a treatment plant designed to process the peak flow of 13.44 mgd (50870 m^3/day). No equalizing storage is used.

b) Construct a storage tank which completely equalizes the flow [1.47 mg. in this case (5564 m^3)] and build a much smaller treatment plant designed to process the equalized flow of 6.88 mgd.

The production function for this approach is simply two points.

Option	T, mgd (m^3/day)	S, mg (m^3)
1	13.44 (50870)	0 (0)
2	6.88 (26041)	1.47 (5564)

However, intermediate possibilities of partial equalization should be explored, a sample of which are indicated below:

Option	T, mgd (m^3/day)	S, mg (m^3)
3	12.96 (49054)	.005 (189)
4	11.52 (43603)	.072 (272)
5	10.08 (38153)	.216 (818)
6	8.64 (32702)	.552 (2090)
7	7.20 (27252)	1.260 (4769)

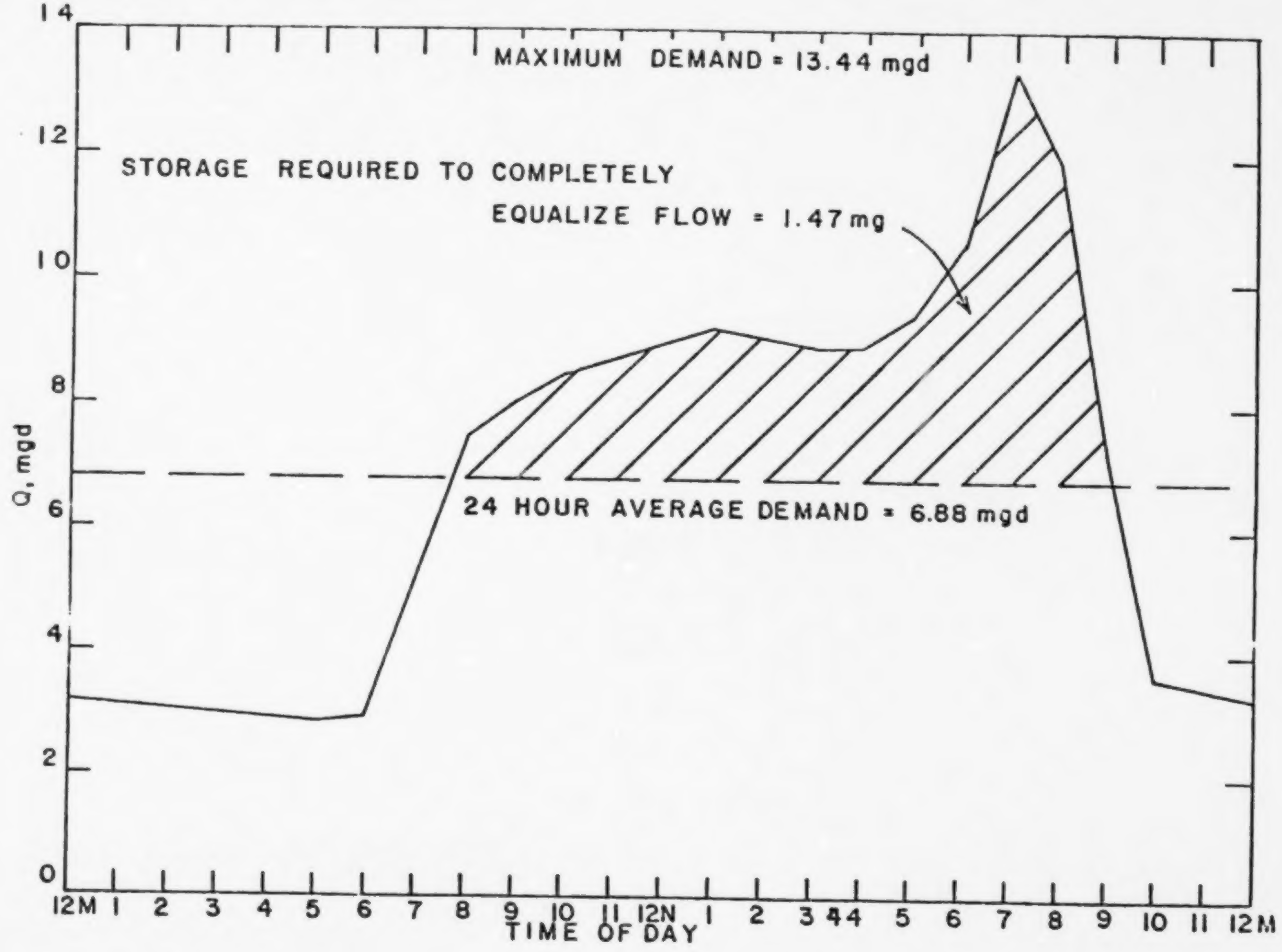


Figure 4-3. Hourly Inflow to Sewage Treatment Plant on the Maximum Day

These options are plotted in Figure 4-4 as a production function with a given level of pollution control as the output, and treatment and storage as the inputs. Typically, the optimal solution will call for partial equalization.

Street Sweeping

Street sweeping may be considered a production process. In this case, the input is the fraction of the days on which sweeping occurs during a year and the output is the fraction of BOD removed.

In order to generate this production function, a model was developed to simulate the conditions within an urbanized area and the effect of street sweeping (1). Hourly rainfall is converted to runoff using a simple runoff coefficient, and subsequently accumulated BOD is removed by scheduled sweeping or a runoff event.

Running the model with several different efficiencies (ranging from 0.1 to 0.9) and sweeping intervals (ranging from 1 to 42 days) generated the family of production functions shown in Figure 4-5. The model was run using data for the developed area of Minneapolis, Minnesota (hourly precipitation for 1971).

Combined Sewer Flushing

Combined sewers often experience dry-weather sewage deposition. These solids accumulate in the sewers until removed by a storm surge or by artificial flushing. The deposition carried away by runoff discharges directly to the receiving water if the dry-weather treatment plant is bypassed. Controlled flushing allows the sewers to be cleaned without adding pollutants to the water body. Instead, these pollutants are routed to the dry-weather plant for treatment.

As with street sweeping, sewer flushing is not a new idea; its primary purpose has been to improve hydraulic capacity and self-scouring ability. However, several reports indicate the ability of a flushing program to remove a substantial portion of the pollutants associated with deposition (15-17).

Pisano provided the data used to develop the sewer flushing production function (15). The input to this process is the fraction of the combined sewer segments flushed daily. The output is the fraction of available BOD removed annually by flushing. The production function is based on information regarding the relative concentration of deposition within the two systems of Boston (e.g., 75 percent of the deposition is found in 18 percent of the Dorchester system elements).

Inspection of the production function shown in Figure 4-6 reveals that sewer flushing also exhibits decreasing marginal output as the input is increased. In fact, the effect is dramatic after approximately 10 or 20 percent of the sewers are flushed daily.

Storage-Treatment

The STORM model was used in an earlier study to generate isoquants of the percent pollutant control, y, as a function of treatment rate, x_1, and

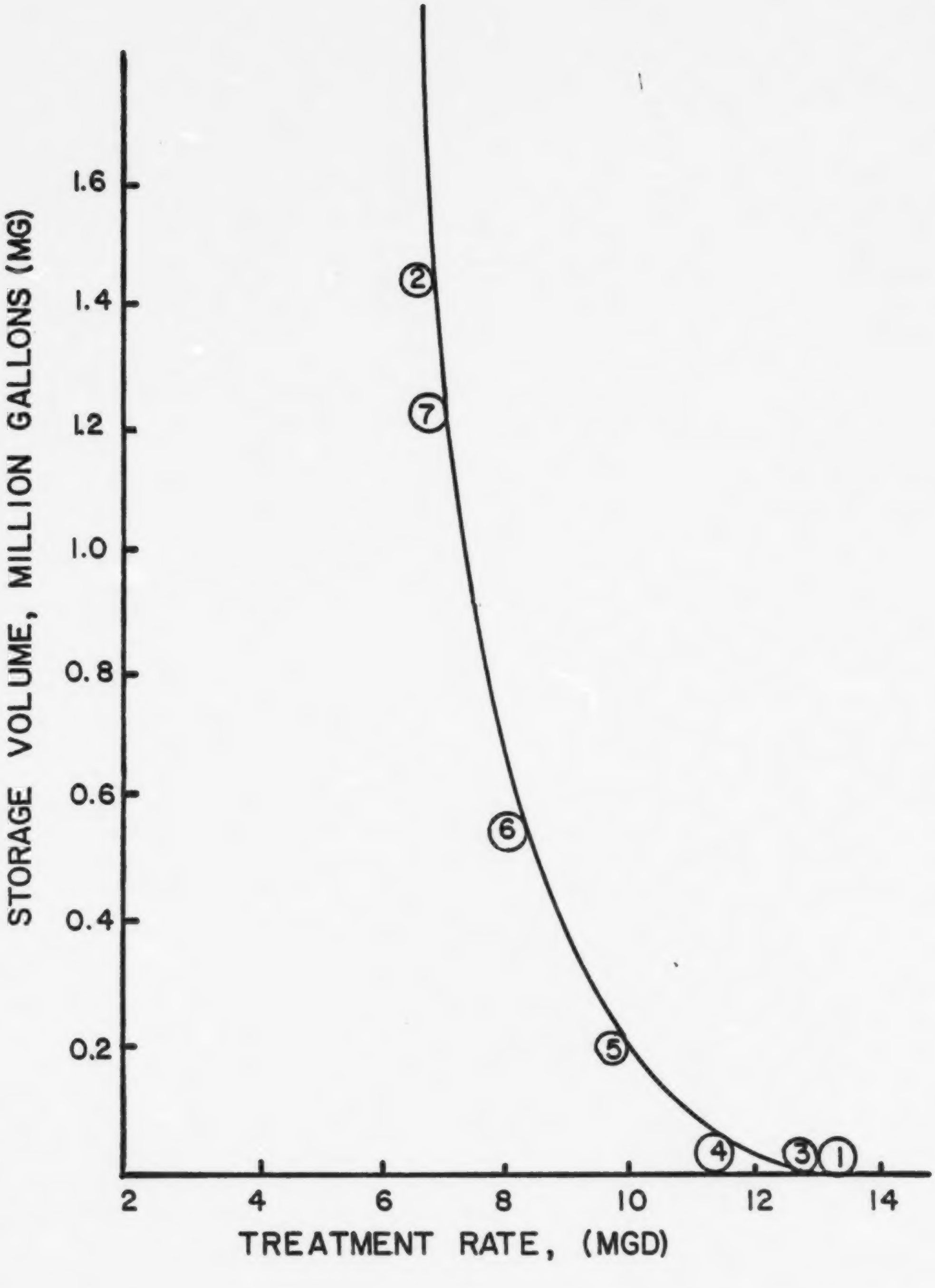


Figure 4-4. Isoquant For Equalizing Storage in Sewage Treatment Plant.

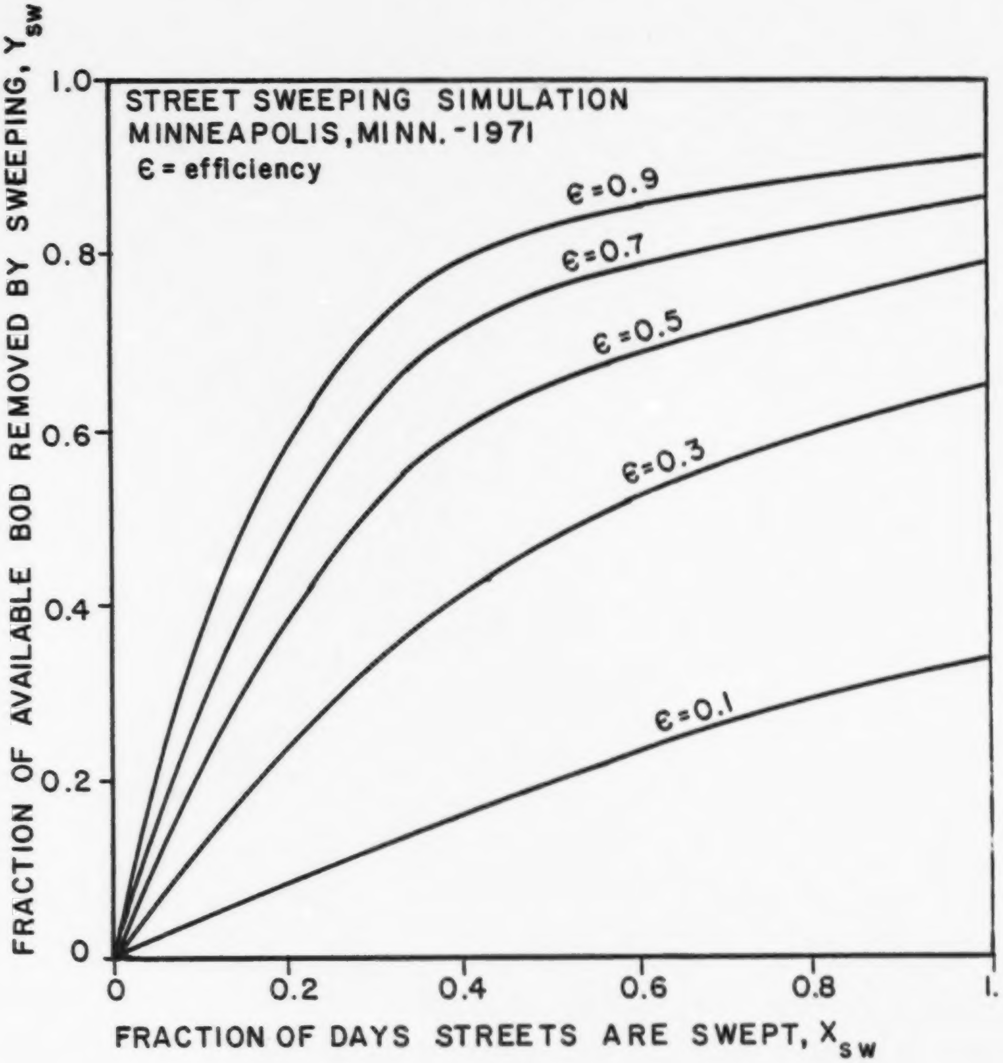


Figure 4-5. Production Functions for Street Sweeping (1).

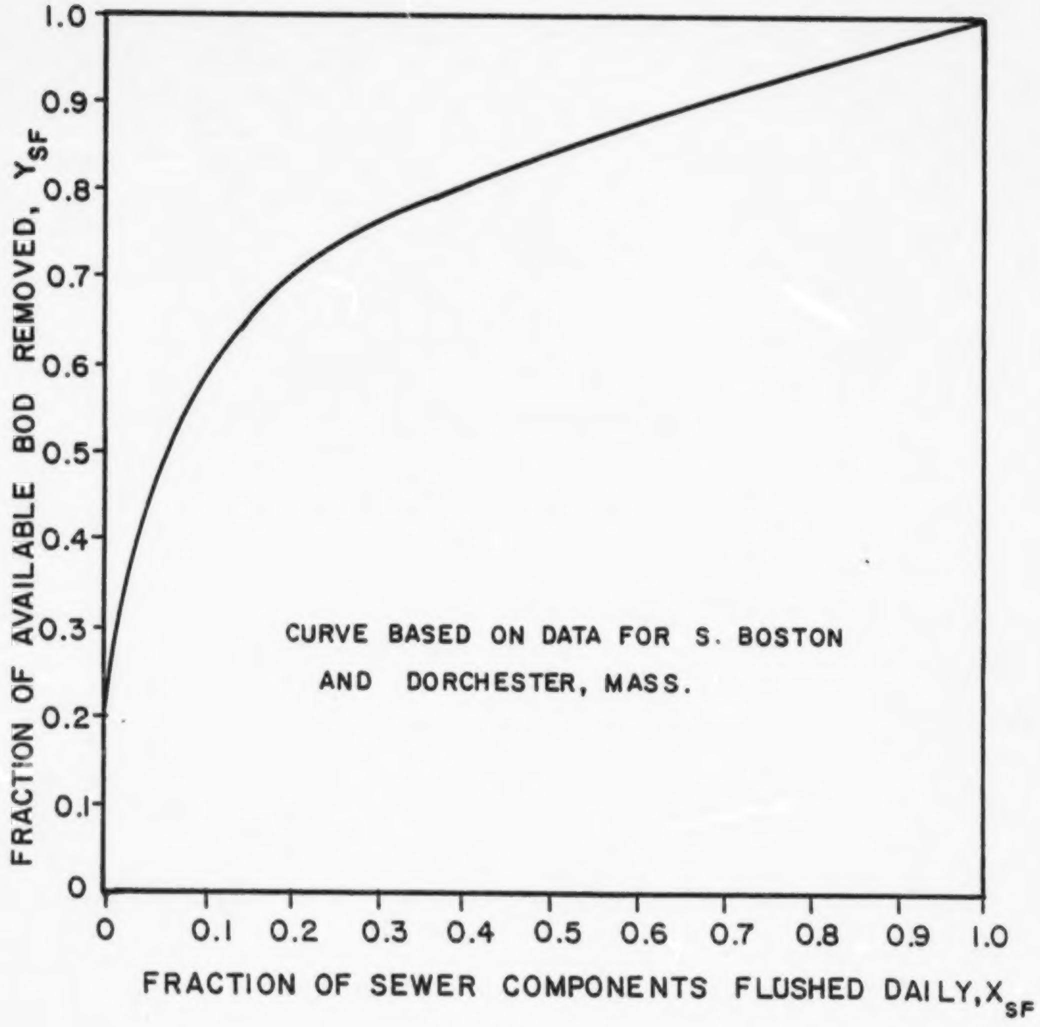


Figure 4-6. Production Function for Combined Sewer Flushing (15).

storage volume, x_2 (18). Results for Minneapolis are shown in Figure 4-7. The isoquants are interpolations between the data points from the simulation runs. Each data point is the result of a one year simulation with a specified storage-treatment configuration. These isoquants exhibit several relevant properties:

1) They slope downward and to the right because as one input increases, it takes less of the other input to achieve the same level of output.
2) They are convex to the origin because of the decreasing ability of one input to be substituted for another to obtain a given level of output. This is known as the principle of diminishing marginal rate of substitution.
3) The isoquants intersect the x_1 axis indicating that it is possible to use this input exclusively.
4) The isoquants become asymptotic to the x_2 axis at the point where x_1 is being used continuously. Further increases in x_2 are nonproductive because x_1 is limitational.

Erosion Control Practices

Chen (19) summarizes the effectiveness of several erosion control practices. For sediment control basins, the trap efficiency is shown in Figure 4-8. The selected measure of effort is the ratio of basin area to outflow rate. However, note that this term is simply the inverse of the overflow velocity. Using the d = 0.016 mm particle and replotting the indicated curve onto arithmetic paper yields the direct production function form shown in Figure 4-9. The function is seen to exhibit diminishing marginal productivity throughout the control range. A second abscissa is added to express the level of effort in the more standard unit of detention time. As the results indicate, the estimated detention times are "reasonable."

Production functions for mulching and for equally spaced diversions, presented by Chen (19), are shown in Figure 4-10. Diminishing marginal productivity is evident in all of these cases.

PROCESS PRODUCTION FUNCTIONS FOR AGRICULTURAL AND INDUSTRIAL ACTIVITIES

All of the above controls were devices whose primary purpose is pollution control. For many activities, process changes can be implemented to reduce the quantity of residuals being discharged. In order to evaluate these process change alternatives, it is necessary to know the overall production function for the agricultural activity (20), e.g., Figure 4-11 or industry (21), e.g., Figure 4-12. Because of the complexity of these activities, it is usually impossible to describe the process options with one or even a few equations. For these more complicated processes, heavy reliance is placed on mathematical programming methods to characterize these production functions. Charnes and Cooper (22) pioneered these applications. If such a model exists for the agricultural activity of concern, it is relative-

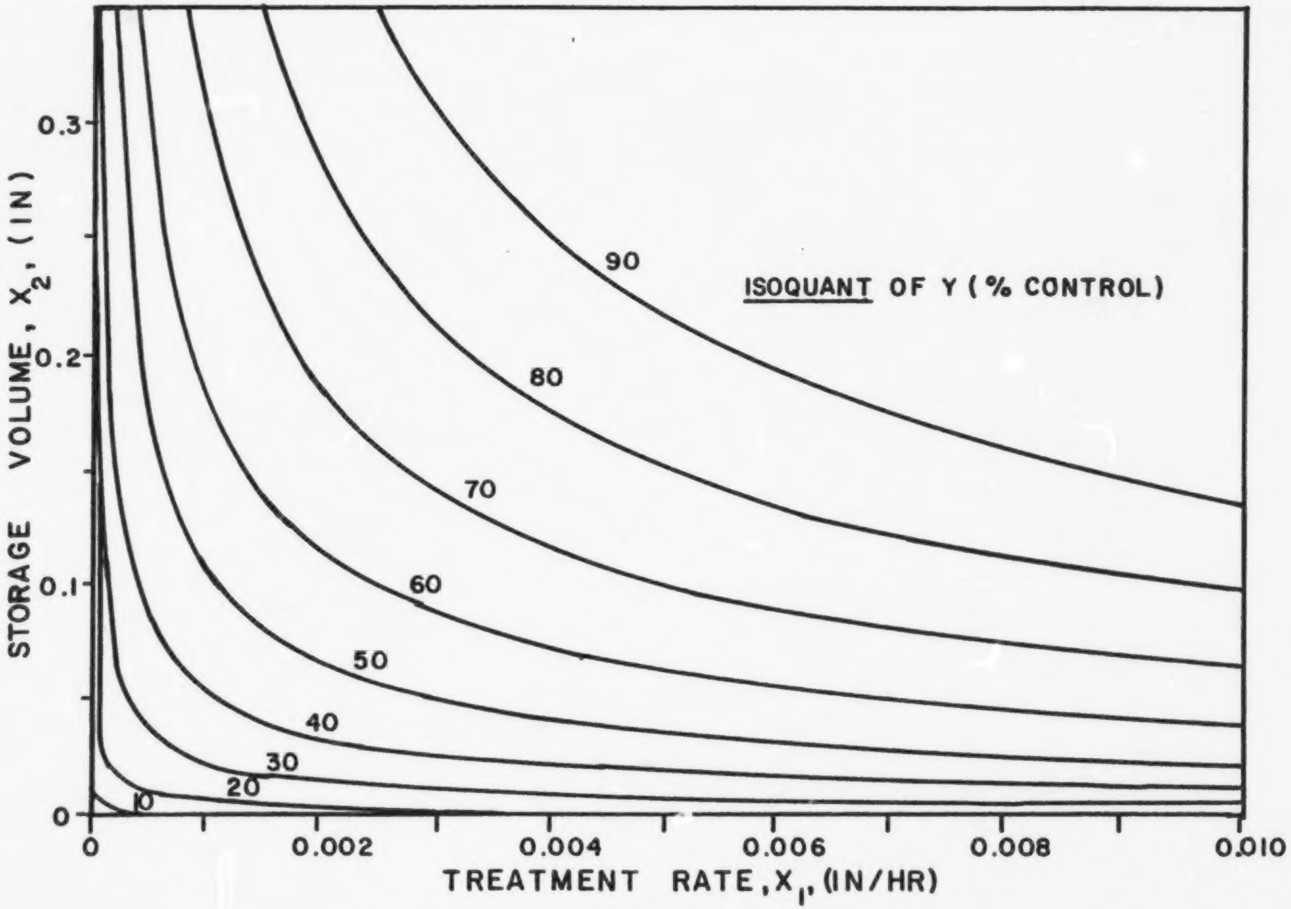


Figure 4-7. Storage-Treatment Isoquants for Minneapolis, Minnesota (18).

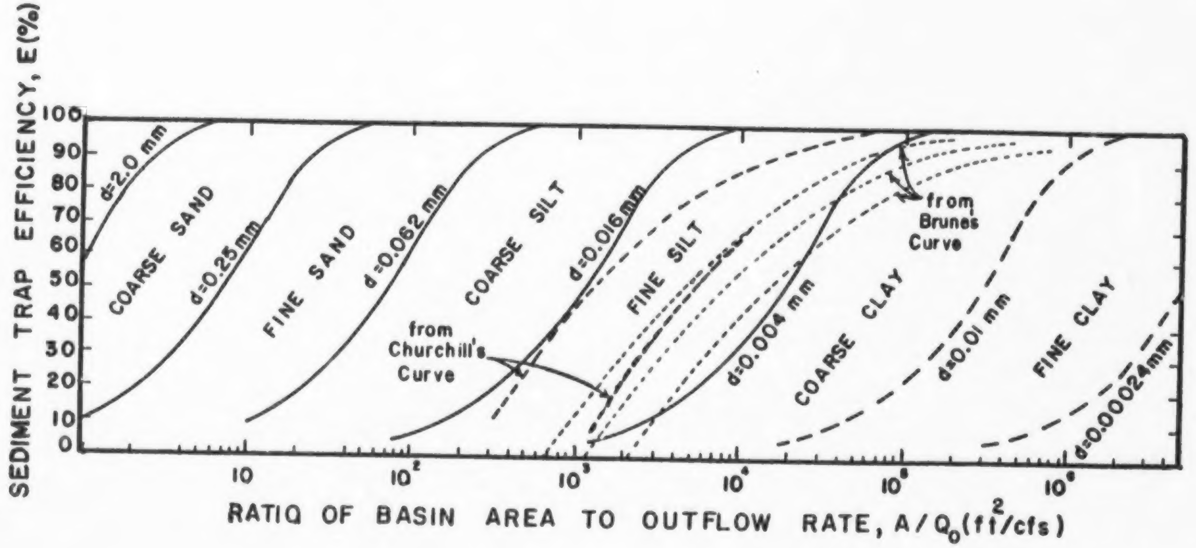


Figure 4-8. Trap Efficiency versus Ratio of Basin Area to Outflow Rate (19).

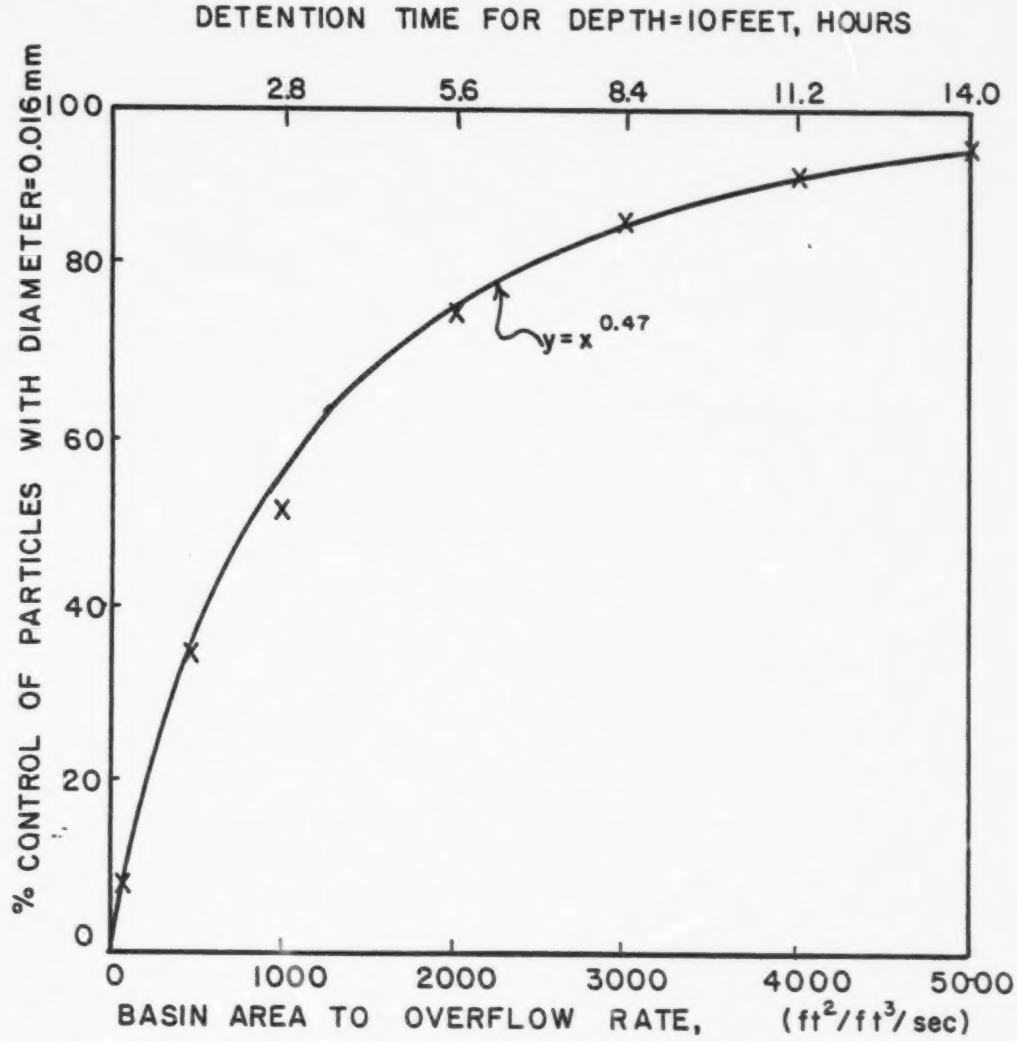


Figure 4-9. Trap Efficiency Curve in Production Function Form.

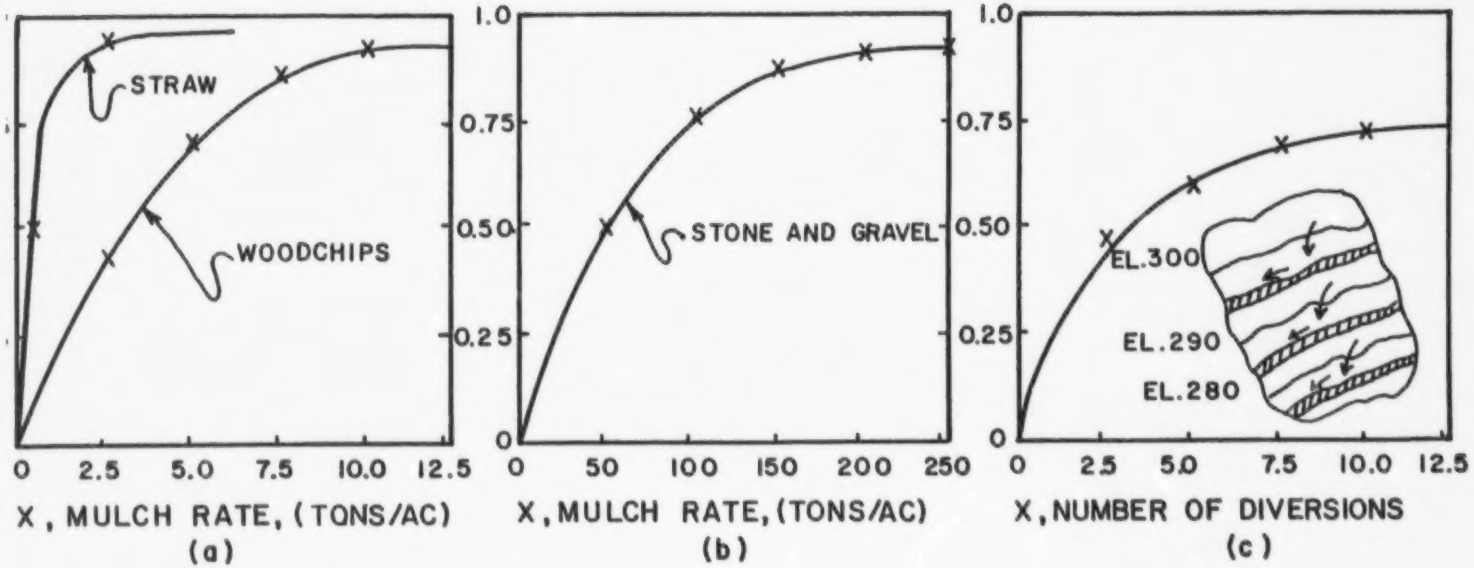


Figure 4-10. Production Functions for Erosion Control, Mulching and for Equally Spaced Diversions (19)

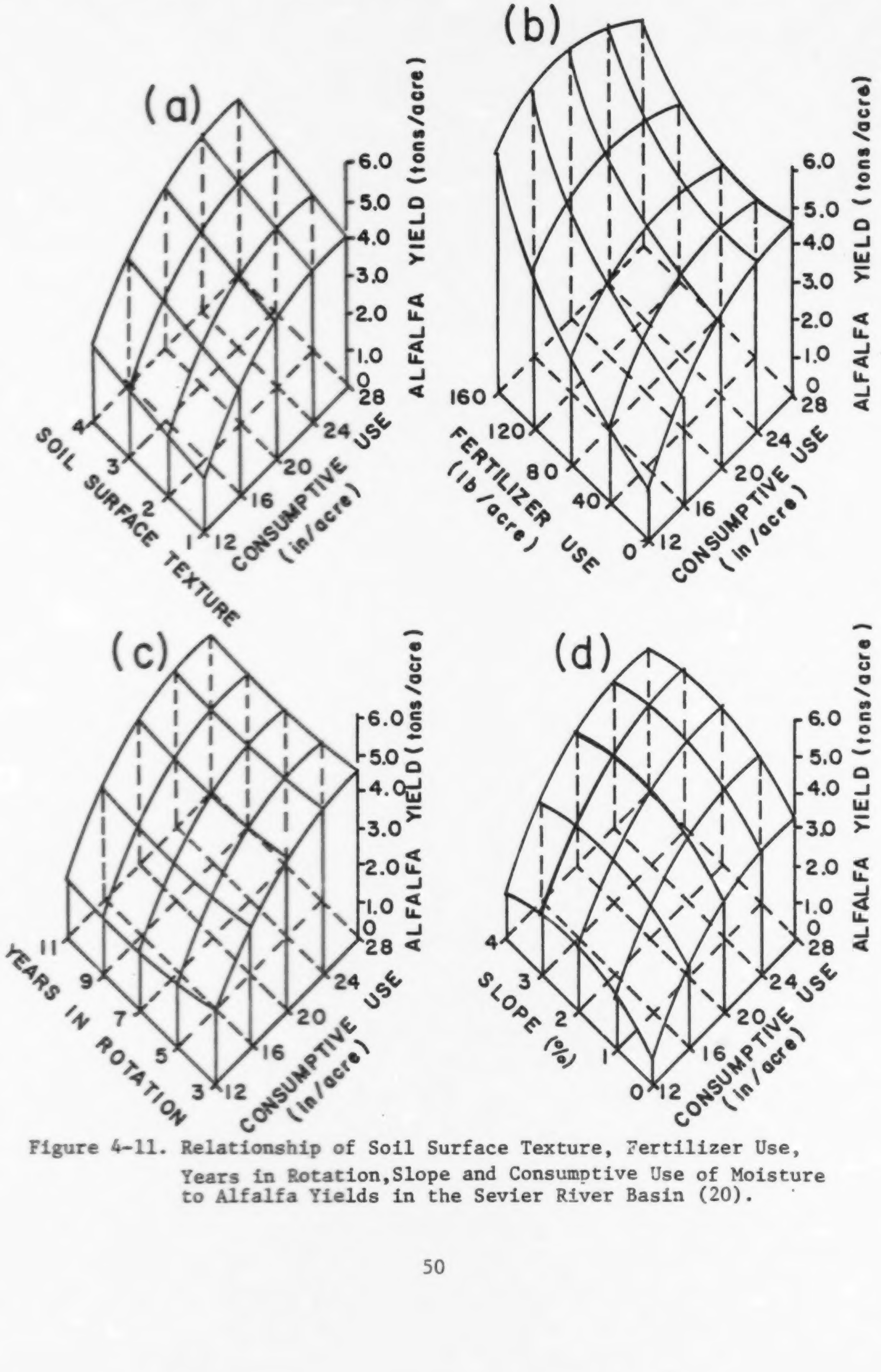


Figure 4-11. Relationship of Soil Surface Texture, Fertilizer Use, Years in Rotation, Slope and Consumptive Use of Moisture to Alfalfa Yields in the Sevier River Basin (20).

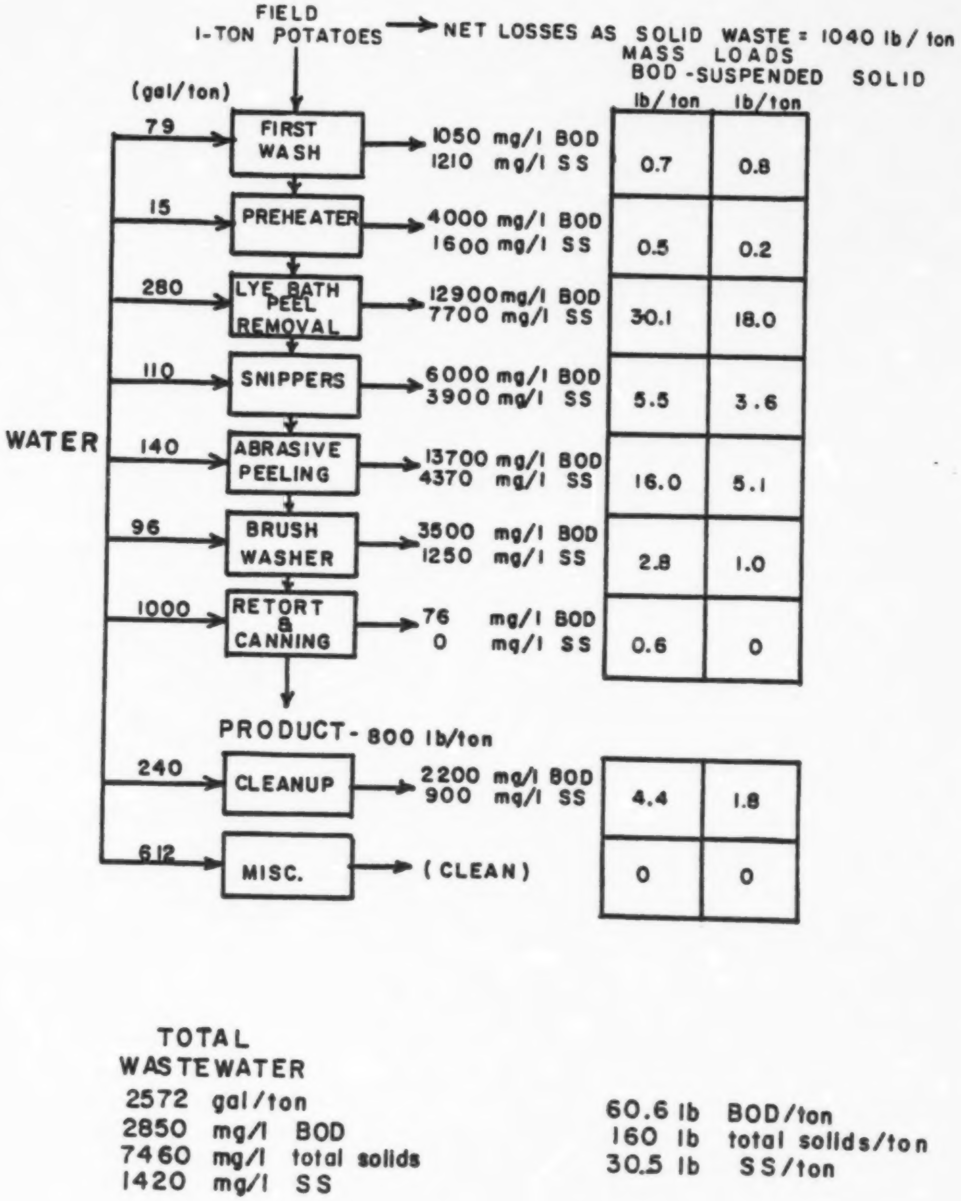

FIELD
1-TON POTATOES
NET LOSSES AS SOLID WASTE = 1040 lb/ton
MASS LOADS
BOD - SUSPENDED SOLID
lb/ton
lb/ton
(gal/ton)
WATER
79
FIRST WASH
1050 mg/l BOD
1210 mg/l SS
0.7
0.8
15
PREHEATER
4000 mg/l BOD
1600 mg/l SS
0.5
0.2
280
LYE BATH PEEL REMOVAL
12900 mg/l BOD
7700 mg/l SS
30.1
18.0
110
SNIPPERS
6000 mg/l BOD
3900 mg/l SS
5.5
3.6
140
ABRASIVE PEELING
13700 mg/l BOD
4370 mg/l SS
16.0
5.1
96
BRUSH WASHER
3500 mg/l BOD
1250 mg/l SS
2.8
1.0
1000
RETORT & CANNING
76 mg/l BOD
0 mg/l SS
0.6
0
PRODUCT - 800 lb/ton
240
CLEANUP
2200 mg/l BOD
900 mg/l SS
4.4
1.8
612
MISC.
(CLEAN)
0
0
TOTAL WASTEWATER
2572 gal/ton
2850 mg/l BOD
7460 mg/l total solids
1420 mg/l SS
60.6 lb BOD/ton
160 lb total solids/ton
30.5 lb SS/ton

Figure 4-12. Sweet Potato Processing-Material Balance on Wastewater (21).

ly simple to add direct pollution control option to this model. Then the optimal mix of modifications in production processes and control options can be determined.

PRODUCTION FUNCTION REPRESENTATION

The preceding section presented a variety of production functions encountered in water pollution control studies. They are obtained from numerous sources and may be presented graphically and/or in functional form.

Graphical Representation

In graphical form, the functions may be plotted on arithmetic, semi-log, or log-log paper. The use of log paper is very common in engineering and is certainly the most convenient form for many applications. Unfortunately, functions plotted on log paper do not show directly whether marginal returns are increasing, decreasing, or both. Thus, it is helpful to replot these results on arithmetic paper to see the shape of the untransformed function. If marginal productivity is ever decreasing then the function bounds a convex set and the subsequent optimization is much more straightforward. On the other hand, if marginal productivity increases initially and then decreases, the function is non-convex. This is the usual shape of the production function shown in economics texts.

For pollution control units, a more general argument can be made that logistic-type curves would not arise. Assume that a control unit has a performance curve as shown in Figure 4-13. For a desired level of control of 27% ($y = 0.27$), a level of effort of 0.40 would be needed. This performance is based on routing the entire wastewater flow through the control units as shown in Figure 4-14. A better procedure is to split the flow, bypassing a portion of it and treating the remaining flow more intensively. In this case, the flow-weighted removal (y) is

$$\begin{aligned} y &= 0 \cdot \alpha + x(1 - \alpha) \qquad (4\text{-}5) \\ &= x(1 - \alpha) \end{aligned}$$

The problem is to maximize the level of control, y. The solution can be found graphically by drawing a ray to the highest point of tangency with the performance curve, point E in this case. The line segment joining the origin and point E gives the level of control that could be attained by diverting from none up to α amount of the flow. Thus, if 28% control is needed, a level of effort of .28 is sufficient, or using equation 4-4,

$$.28 = .66(1 - \alpha).$$

Thus, 58% of the flow ($\alpha = .58$) should bypass the control unit. The remaining 42% of the flow is treated at a level of effort of 0.66 which yields an overall weighted average control level of

$$y = 0.58(0) + .66(.42) = .28$$

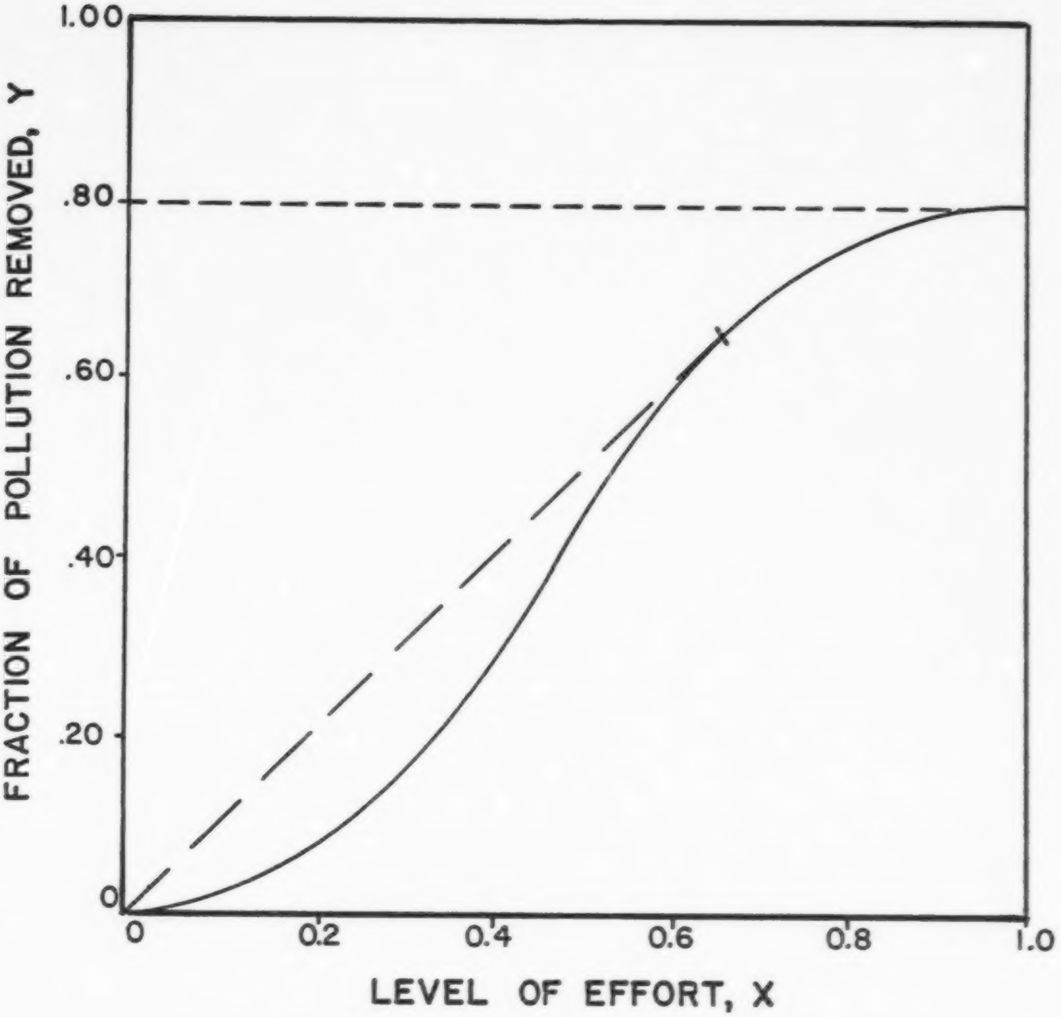


Figure 4-13. Hypothetical Production Functions for Control Unit.

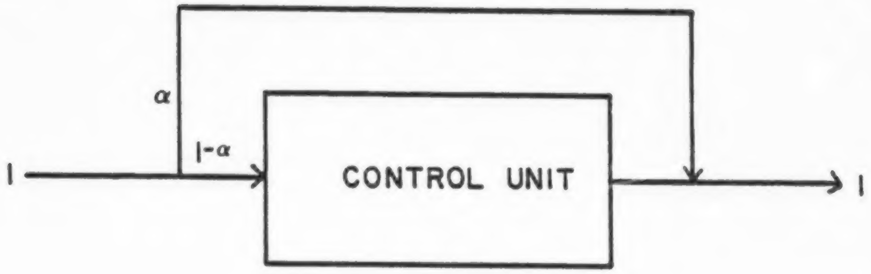

α
1-α
1
CONTROL UNIT
1

Figure 4-14. Alternative Flow Pattern Through Control Unit.

as desired. Thus, the correct production function is the ray to point E and follows the curve beyond that point. This revised function does bound a convex set.

Functional Representation

The usual method of curve fitting in engineering yields the functional form which "best" (usually in a least squares sense) fits the available data. Unfortunately, the selected functional form is often relatively unwieldy to manipulate in the associated economic and financial analysis. Furthermore, the selected measures of level of effort may not be the desired ones for the remaining analysis. This was pointed out in the aeration tank example presented earlier.

Recall the storage-treatment production functions which were developed for five cities. In order to extrapolate these results to the other 243 urbanized areas, regional functions were developed based on finding the best functional fit. The resulting equation, for a fixed value of y, is (18)

$$x_1 = \underline{x}_1 + (\bar{x}_1 - \underline{x}_1)e^{-kx_2} \qquad (4\text{-}6)$$

where $\underline{x}_1$ = treatment rate at which the isoquant becomes asymptotic to the ordinate, inches/hour.

$\bar{x}_1$ = treatment rate at which isoquant intersects the abscissa, inches/hour, and

k = constant, inch^{-1}.

The lower limit on treatment, $\underline{x}_1$, can be found directly, as follows, for 8760 hours per year:

$$\underline{x}_1 = \frac{AR}{8760}(y) = ay \qquad (4\text{-}7)$$

where AR = annual runoff, inches/year, and

a = coefficient.

By relating the parameters $\underline{x}_1$, $\bar{x}_1 - \underline{x}_1$, and k to the level of control, y, one equation was developed for the entire response surface. The $\bar{x}_1 - \underline{x}_1$ and k terms were found to be of the following general form:

$$\bar{x}_1 - \underline{x}_1 = be^{hy}, \text{ and} \qquad (4\text{-}8)$$

$$k = de^{-fy} \qquad (4\text{-}9)$$

where b, d, f, h = coefficients.

Substituting equations 4-7, 4-8, and 4-9 into equation 4-6 yields the production function:

$$x_1 = ay + be^{hy} - (de^{-fy})x_2 \qquad (4\text{-}10)$$

This function, while statistically the most accurate, proved clumsy to manipulate, e.g., finding $\partial y/\partial x_i$, the marginal productivity of each input, in the subsequent optimization. Thus, another functional form was evaluated.

...Cobb-Douglas Production Function

While a wide variety of production functions are used by economists, this study examines only the so-called Cobb-Douglas production function of one output and two inputs (5). It has the following form:

$$y = Ax_1^{\alpha_1} x_2^{\alpha_2} \qquad (4\text{-}11)$$

where A = coefficient, and

α_1, α_2 = input intensity parameters, $\alpha_1 < 1$, $\alpha_2 < 1$.

This function has several properties which make it convenient to use (8) e.g.,

1) it is a homogeneous equation;

2) if $\alpha_1 + \alpha_2$
   - <1 → decreasing returns to scale
   - =1 → constant returns of scale
   - >1 → economies of scale;

3) marginal products, $\partial y/\partial x_i$, are positive but decreasing as x_i increases; and

4) the optimal solution for any value of y can be expressed directly as a function of the inputs.

A potentially major limitation of this functional form is that it is asymptotic to both input axes. Thus, it does not permit using treatment, x_1, alone. This problem can be avoided by defining another single-input production function,

$$y = A_1 x_1^{\alpha} \qquad (4\text{-}12)$$

where A_1 = coefficient, and

α = input intensity parameter,

which is used in the optimization procedure as an alternative to equation 4-11. Comparison of the results will tell the range over which each production function is superior. Another limitation of the Cobb-Douglas functional form is that it loses accuracy as one approaches the x_2 asymptote. However, experience to date indicates that the "elbow" of the function is the area of primary concern.

In the earlier work, isoquants (using the exponential function) were generated for San Francisco, Denver, Minneapolis, Atlanta, and Washington, D.C. (18). Table 4-5 shows the results of fitting two-input and single-input production functions to this same data. The sum of the α's is less than one, indicating diseconomies of scale. Lastly, a single predictive equation for the entire United States was determined by using the data for the five cities with annual runoff as the third input. The result of the two-input model is:

$$y = [967\ (AR)^{-0.239}]\ x_1^{0.233}\ x_2^{0.390} \qquad (4\text{-}13)$$

A comparison of the accuracy of the Cobb-Douglas model versus the more refined ones will be presented in the section on economic analysis.

...Mathematical Programming

For larger problems it is usually necessary to represent production functions in a form suitable for a linear programming model. For simple production functions, i.e., $y = f(x)$, the nonlinear relationships can be represented, to any desired accuracy, by piecewise linear approximations as shown in Figure 4-15. As long as the production function bounds a convex set, i.e., the line joining any two points on the boundary of the set lies within the set, then this procedure guarantees a globally optimal solution. The additional effort to do piecewise approximations is not great and the bounded variables do not count as additional constraints on the larger linear programming codes such as IBM's MPS.

Similar linearization methods can be used for more complicated production functions. For example, the storage-treatment problem deals with two inputs (storage and treatment) and one output, pollution control. This function can be represented in equivalent linearized form in two ways:

(a) Piecewise linear approximation of single isoquant when desired level of control is known.

The solution, shown in Figure 4-16, is to replace the nonlinear isoquant by piecewise linear segments, four in this example. The linear programming formulation of this problem is to

$$\text{minimize } Z = c_S S + c_T T$$

subject to

$$\begin{aligned} T &\geqq T_0 \\ S + (S_1/T_1)T &\geqq T_1 \\ S + (S_2/T_2)T &\geqq T_2 \\ S + (S_3/T_3)T &\geqq T_3 \\ S,T &\geqq 0 \end{aligned} \qquad (4\text{-}14)$$

TABLE 4-5. Parameters for Cobb-Douglas Production Function for Five Cities

City	j=	Annual Runoff,[a] in Inches per Year AR	Developed Population Density,[a] in Persons per Acre	Cobb-Douglas Production Function Coefficients				
				$y^b = A\,x_1^{\alpha_1}\,x_2^{\alpha_2}$			$y^c = A_1\,x_1^{\alpha}$	
				A	α_1	α_2	A_1	α
San Francisco	1	9.37	9.96	489	0.198	0.410	566	0.65
Denver	2	5.59	9.11	584	0.252	0.334	449	0.56
Minneapolis	3	10.50	7.92	464	0.213	0.333	410	0.56
Atlanta	4	16.18	8.24	660	0.246	0.490	481	0.69
Washington, D.C.	5	17.22	20.02	570	0.255	0.394	440	0.63

[a] Source: Reference 18.

[b] Multiple correlation coefficients ≧ .95.

[c] Correlation coefficients ≧ 0.92.

$$
\begin{aligned}
a_1x_1 + a_2x_2 + a_3x_3 &= y \\
x_1 &\le \bar{x}_1 \\
x_2 &\le \bar{x}_2 \\
x_3 &\le \bar{x}_3
\end{aligned}
$$

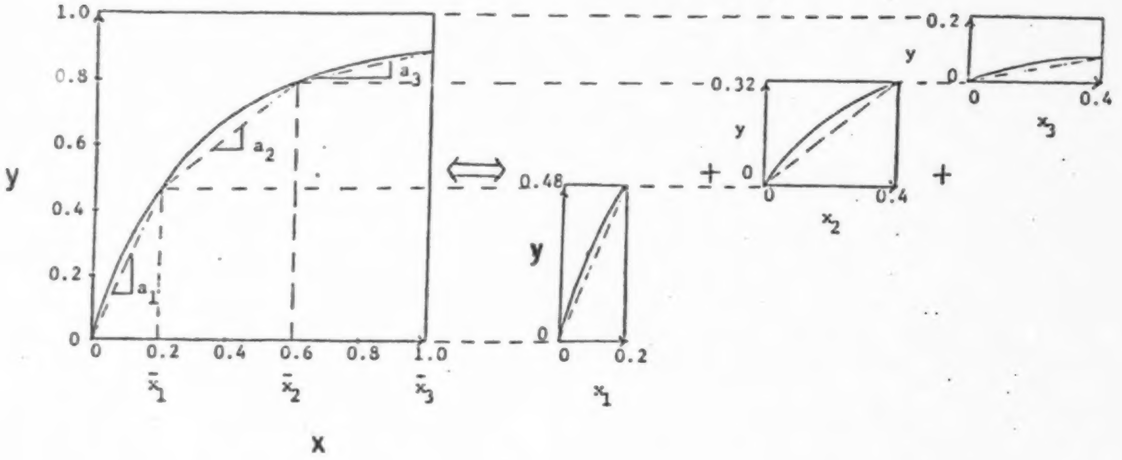


Figure 4-15. Piecewise Linearization of Nonlinear Production Function.

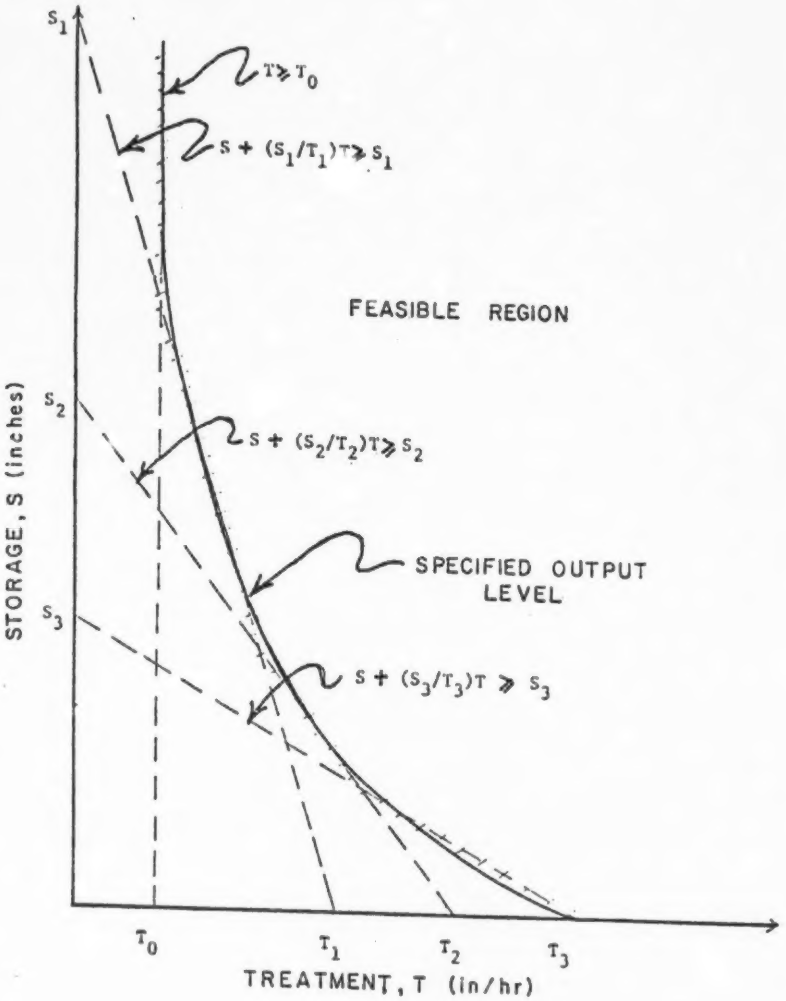

S1
T ≥ T0
S + (S1/T1)T ≥ S1
FEASIBLE REGION
S2
S + (S2/T2)T ≥ S2
STORAGE, S (inches)
SPECIFIED OUTPUT LEVEL
S3
S + (S3/T3)T ≥ S3
T0
T1
T2
T3
TREATMENT, T (in/hr)

Figure 4-16. Piecewise Linearization of a Single Isoquant.

Since this is a minimization problem, the feasible region is bounded from below by the crosshatched line segments shown in Figure 4-16. Any desired level of accuracy can be obtained simply by adding additional lines (each one of which adds an additional constraint) to the linear programming problem.

If the level of output is part of the decision variables, the above procedure becomes unwieldy. Fortunately, a relatively straightforward approach exists (23).

Recall the storage-treatment production function presented earlier. This response surface can be represented in mathematical programming form as a series of "processes," P_1, P_2, ..., P_k, as shown in Figure 4-17. Then, a "process" is simply any one of these rays. For example, process 3, a fixed mix of treatment and storage, is represented by

$$t_3 T - S = 0 \tag{4-15}$$

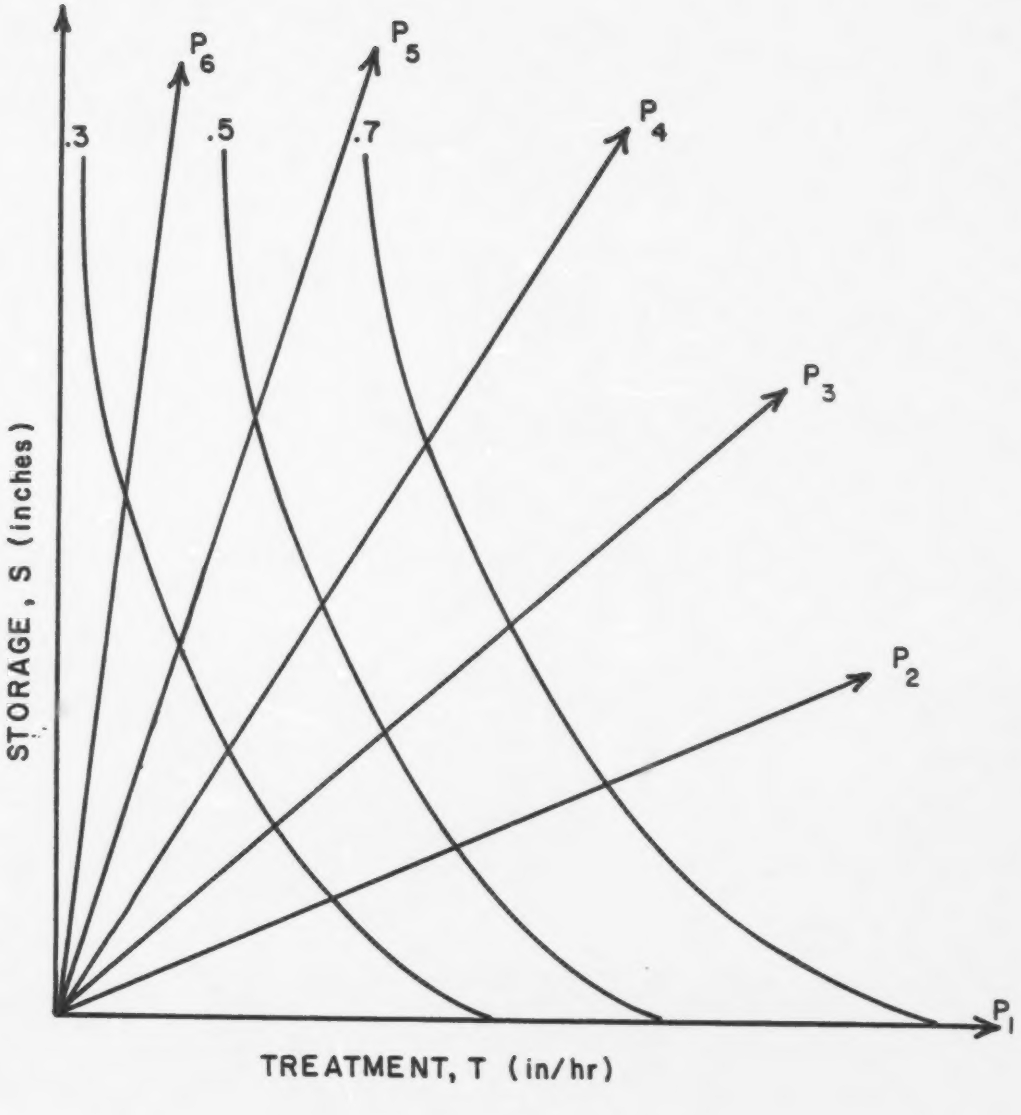
In this example, t_i has a very specific meaning. From equation 4-14,

$$t_i = S/T. \tag{4-16}$$

But, the units of t_i are simply time. Indeed, t_i is the time to drain the full reservoir with a capacity of S inches at a rate of T inches/hour. Accordingly, P_1 has a zero drawdown time, P_2 has a drawdown time of 6 hours, all the way up to P_6 with a drawdown time of 500 hours. As before, each of these processes may have a nonlinear response function. The physical analog is to assume that these options are in parallel as shown in Figure 4-18. Knowing these production functions and the unit costs of each of these "processes," the LP model selects the optimal mix.

CONCLUSIONS

This chapter presented an approach for representing the technological aspects of the decision making problem using production functions. This method permits engineers and economists to interact more effectively in evaluating alternatives. The production function representation provides a concise and relatively easy to understand way to depict the very important technological aspects of the problem. An auxillary objective was to provide an appreciation of the need to develop these functions very carefully and consistently when comparing the wide variety of control options. The selected functions should undergo careful assessment and be approved by the designers and operators of the controls.



STORAGE, S (inches)
TREATMENT, T (in/hr)
.3
.5
.7
P6
P5
P4
P3
P2
P1

Figure 4-17. Approximation of Production Function as a Discrete Number of Processes.

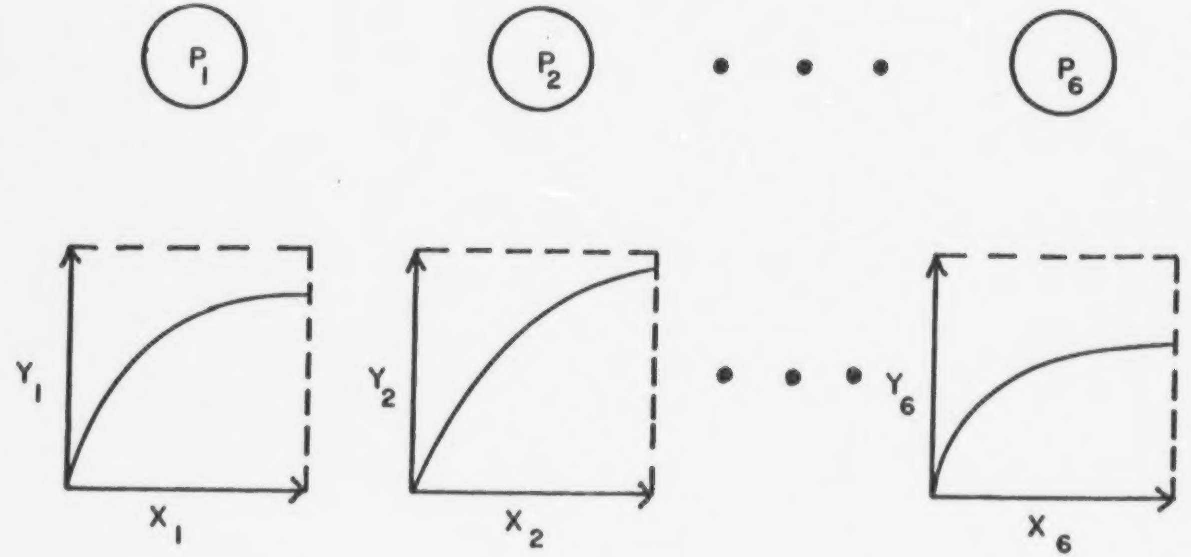

P_1
P_2
P_6
Y_1
X_1
Y_2
X_2
Y_6
X_6

Figure 4-18. Representation of Processes as Options in Parallel.

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CHAPTER 5

ECONOMIC ANALYSIS OF CONTROL ALTERNATIVES

INTRODUCTION

Benefit-cost analysis seeks to determine the level of investment which maximizes the positive difference between benefits and costs. General information on this subject is available in references 1 - 6. Applications to water resources problems are described in references 7 - 12. A comprehensive summary of applications to water pollution control has been prepared by Tihansky (13). Interest in benefit-cost analysis as applied to water pollution has been rekindled by an EPA program requirements memorandum (No. 75-34) requiring that a benefit-cost type of analysis be performed as part of combined sewer overflow grant applications to the EPA.

The previous two chapters described the technological aspects of the problem using production functions, and the benefits or measures of effectiveness that can be associated with such control programs. These results indicate that it is very difficult to measure benefits directly because many of the effects of water pollution cannot be readily converted to a monetary equivalent, e.g., the value of a human life. Thus, the common practice is to specify a performance standard, or measure of effectiveness. In this context, the objective of the cost-effectiveness analysis is to find the least costly way to meet this standard.

It is very important to be fully aware that selecting a measure of effectiveness is equivalent to making the social choice of the desired level of environmental quality. This selected level *should* *not* be determined without an associated cost analysis. The nationwide assessment of the cost of controlling storm water pollution vividly illustrates the need to determine the cost implication of performance standards (14). The next section summarizes this effort to set the tone for the rest of the chapter. Then some basic definitions related to cost analysis are presented. The main portion of the chapter deals with straightforward optimization procedures applied to several problems of increasing complexity.

EXAMPLE OF COST-EFFECTIVENESS ANALYSIS

Early studies indicated that urban stormwater control for the United States is quite expensive. The 1974 Needs Survey (15) estimated total control costs as $266 billion whereas the later National Commission on Water Quality (16) estimate was $279 billion. Concurrent cost-effectiveness analysis by Heaney et al. indicated costs ranging from 2.5 billion for 25 percent control to 23.0 billion for 75 percent control, a small fraction of the above estimates (14). About $84 billion of the difference was attributable to the first two studies including storm sewer construction in their estimates. However, subtracting this amount still leaves quite a disparity in the estimates. Further analysis indicated that the larger difference was attributable to the fact that they had used a two year, one hour design storm for their calculations. The results of a sensitivity analysis of control cost vs. percent BOD removal for various design storms, shown in Figure 5-1, are striking! Using the usually accepted criterion of controlling the one, two or five year design storm results in relatively large costs because it is in the range of nearly total capture of the stormwater. A more "reasonable" solution would be to pick a level of control near the "knee" of the curve, i.e., the two week or one month points. The marginal costs of controlling beyond this level are extremely high. Extrapolating this optimized strategy to control of the two year storm yields an estimated control cost of about $100 billion. This estimate is still below the other estimates. The primary reason for presenting these results is to stress the importance of showing cost-effectiveness curves to decision makers before decisions are made as to the "appropriate" design criteria.

GROUND RULES FOR COST ANALYSIS

The cost of a resource, in an economic analysis, is the decrease in wealth which results from committing this resource to a particular use. It measures actual opportunities foregone and hence is referred to as its opportunity cost. A related notion is called the marginal cost, MC, of using a unit of a resource, x_i, where

$$MC_i = \frac{\partial(\text{total cost})}{\partial x_i} \tag{5-1}$$

i.e., it is the partial derivative of total cost with respect to x. Total cost (TC) is the sum of fixed costs (FC) and variable costs (VC), i.e.,

$$TC = FC + VC \tag{5-2}$$

For any decision problem, only variable costs are relevant from an economic point of view. Fixed costs are irrelevant in that they have to be paid irrespective of which option is selected. Unfortunately, in actual applications, it is not always clear which costs are fixed and which are variable.

The cost estimating problem is further complicated by the fact that "costs" are often determined by accountants who have estimated these "costs" for purposes other than decision making, e.g., income taxes. For example, a piece of equipment may be written off or depreciated over a three year period for income tax purposes even though its useful service life is six years. A related problem deals with joint or overhead costs, i.e., costs chargeable to more than one activity. For example, a university library services undergraduate education, graduate education, sponsored research, and unsponsored research. The "costs" of any of these four functions is heavily dependent on the cost sharing rule employed. When dealing with these concepts from

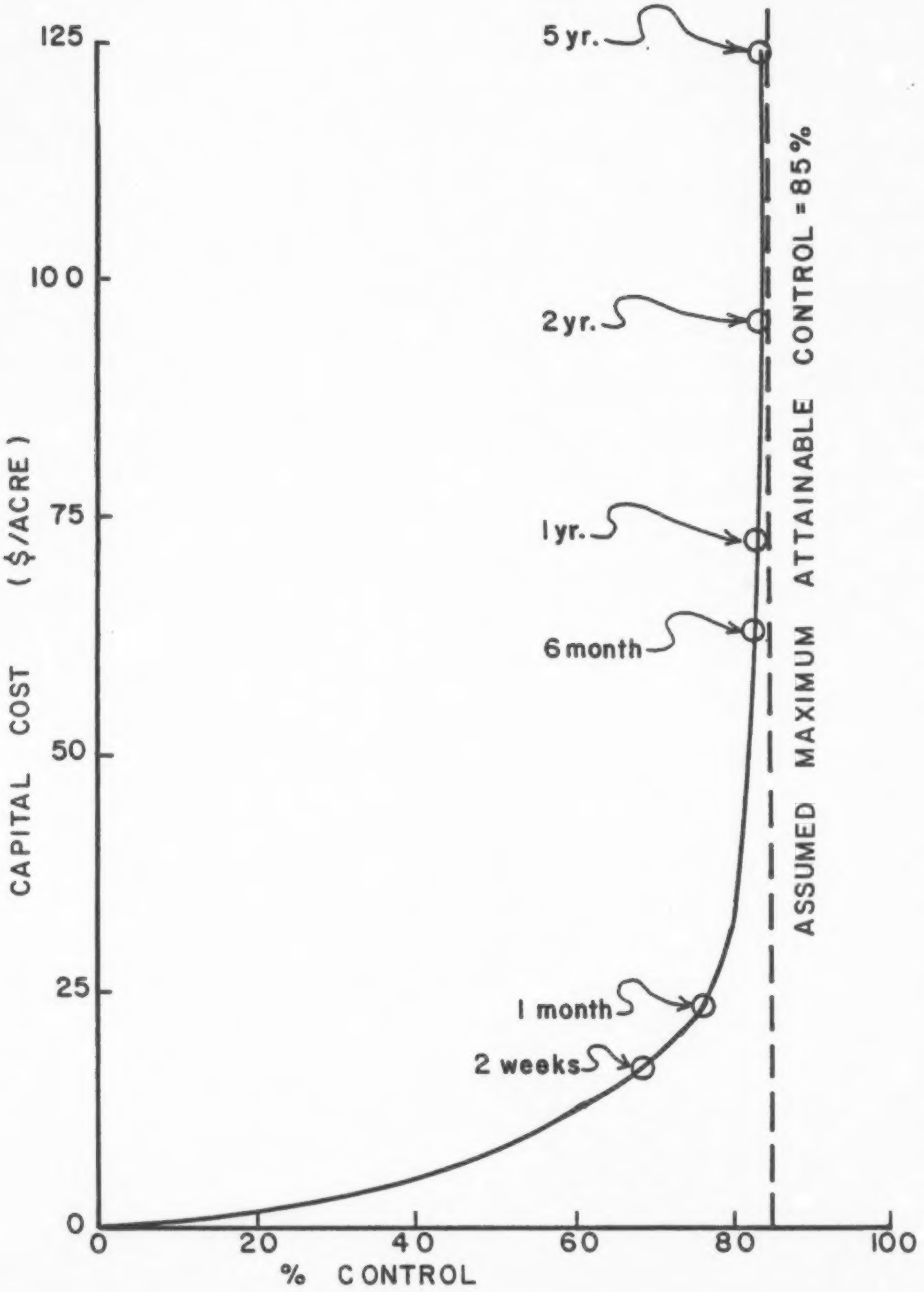

Figure 5-1. Nationwide Cost of Urban Stormwater Pollution Control for Various Design Storms.

accounting, economics, and engineering, it is often quite confusing as to how to properly measure cost. A major source of confusion emanates from the seeming conflict between the economist's insistence on using only marginal costs without regard to whether total costs are covered versus the accountant's insistence on using average costs so that full costs are assigned.

As an example, a recent national survey of street sweeping costs conducted by the American Public Works Association revealed a range of reported values from \$0.28 to \$28,000 per curb mile of street swept (18). This assumed unit cost is the primary driving force in determining the relative effectiveness of street sweepers. Why do they vary over five orders of magnitude? The answer lies in the accounting system used to report unit costs. At the low end, a unit cost of \$0.28 per curb mile would include only very direct sweeper operating costs such as gas and oil. At the other end of the spectrum, the \$28,000 figure would include all operating costs directly attributable to the sweeper, amortized capital costs of the equipment (no doubt over a very short period of time in this case), and some share of overhead costs from the Public Works Department, the mayor's office, etc. Thus, it is essential to agree on bookkeeping methods before doing the cost analysis.

The main body of literature in water resources and environmental management has been developed by engineers and economists. Engineers, guided by principles from economics, conduct benefit-cost analysis and cost allocation studies. The economic analysis stresses the need to examine marginal, rather than average, benefits and costs. This approach may cause difficulty if a significant portion of the benefits and costs remain unallocated. Economic theory has also had a significant impact on cost accounting. Thus, the cost accounting literature was reviewed to gain insight as to how accountants have applied economic theory and whether they have found it to be "practical". The material used in this section is drawn from Horngren (20), Matz and Usry (21), and Demski and Feltham (22).

With regard to the marginal cost vs. average cost controversy, the analogue in accounting is called direct vs. absorption costing. Direct costing is the same as marginal or variable costing. Absorption costing is the widely accepted practice in the accounting field. Direct costing is seen to have a significant impact as a guide for internal decision making. However, it is not an accepted way to report costs for external purposes, e.g., for taxes or asset valuation (18). The marginal vs. average cost question arises in setting rates for utility service, short-term use of "excess" capacity as in the sewage treatment plant/pipeline example, finding the unit cost of street sweeping which also has non-pollution control purposes, etc. No "correct" rule appears to exist. The marginal cost principle appears to be more relevant in the short-run, i.e., weeks or months. The full absorption cost procedure is more appropriate for the long run analysis of major alternatives - the prime concern of this study.

Accountants have devised procedures for dealing with the related problem of determining the cost of joint products. In water pollution, a waste treatment plant produces the joint products of simultaneously removing several pollutants, e.g., BOD, SS, N, and P. The environmental decision maker may wish to know the cost of removing one pollutant, say phosphorous. However, its cost depends on whether it is viewed as a primary product or a by-product. Primary products are the joint outputs of relatively large value. Thus, the definitions may change over time. A joint product example will be presented later.

From an accountant's point of view, costs are allocated for several purposes, including: (1) predicting the economic effect of decisions, (2) determining income and valuing inventory, (3) ascertaining a mutually agreeable price, and/or (4) obtaining the desired motivation in management (20). Because of this multiplicity of objectives of cost allocation, the appropriate procedure must be selected on a case-by-case basis. For our purposes, the general objective is to evaluate the long-term economic effect of decisions. With regard to costing, Horngren (20) concludes that "where feasible, fixed costs should be reallocated by using predetermined monthly lump sums for providing a basic capacity to serve. Variable costs should be reallocated by using a predetermined standard unit rate for the services utilized."

The balance of this chapter on cost-effectiveness assumes that some agreed upon method of costing has been adopted. The next chapter on cost allocation will examine this problem in detail.

The reader is assumed to be familar with basic notions of engineering economics as outlined in standard texts on the subject. Annualized costs will be used throughout the analysis.

The problem is to find the combination of all of the above control options which achieves a specified pollutant control level at minimum cost. As part of earlier EPA sponsored studies, such procedures have been developed (14, 23, 24). The method can be expressed as a problem in production economics, i.e.,

$$\begin{aligned} &\text{minimize } Z = f(\bar{X}) \\ &\text{subject to } g(\bar{Y},\bar{X}) = 0 \\ &\qquad\qquad \bar{Y} \geqq \hat{Y} \end{aligned} \qquad (5\text{-}3)$$

where Z = cost of attaining a specified output level;

$f(\bar{X})$ = cost function for a vector of inputs, $\bar{X}$;

$\bar{X}$ = vector of inputs, $\bar{X} = (x_1, x_2, \ldots, x_i, \ldots, x_m)$;

$\bar{Y}$ = vector of outputs, $\bar{Y} = (y_1, y_2, \ldots, y_j, \ldots, y_n)$;

$\hat{Y}$ = specified output level, $\hat{Y} = (\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_j, \ldots, \hat{y}_n)$;

and

$g(\bar{Y},\bar{X}) = 0$ is a production function expressing the maximum attainable level of output which can be achieved with a given vector of inputs, $\bar{X}$.

This rather esoteric formulation can be translated into a relevant engineering tool if appropriate cost data and knowledge of the production functions are available. Production functions presented in the previous chapter are combined with cost functions and a pre-specified performance standard to find the optimal solution. The remaining sections of this chapter present applications of increasing complexity.

SIMPLE WASTEWATER TREATMENT OPTIMIZATION

Single Plant With Specified Standard

Production functions for wastewater treatment systems presented in Table 4-4 are repeated here as Table 5-1. They are of the simple form:

$$y_j = a_{ij}\, x_i \tag{5-4}$$

where y_j = proportion of pollutant j removed $(0 \le y_j \le 1.0)$

x_i = control i, $x_i = 0,1$

a_{ij} = proportion of pollutant j removed per unit of x_i.

The total annual cost of each of the twelve controls is expressed using a single functional form,

$$C_i = d_i\, Q^{b_i} \tag{5-5}$$

where C_i = total annual cost of $i^{\underline{th}}$ control, \$/yr.

Q = plant flow, mgd, and

d_i, b_i = parameters.

Using this functional form is convenient since it is possible to tell, by inspection, whether economies of scale are present. If

$$b_i \quad \begin{array}{l} > 1 \rightarrow \text{diseconomies of scale} \\ = 1 \rightarrow \text{constant returns to scale} \\ < 1 \rightarrow \text{economies of scale} \end{array}$$

The b_i values, shown in the last column of Table 5-1, are all less than one, typically being about 0.7. Thus, from the point of view of treatment, there may be an incentive to build larger units.

The economic optimization problem for this case is to

Table 5-1. Wastewater Treatment System Production and Cost Function[1]
Values of a in $y_j = a_{ij}\ x_i\ (x_i = 0,1)$

System Number i =	1 BOD_5	2 TSS	3 COD	4 Total P	5 NH_3-N	6 UOD[2]	Description of System	Annual Cost[3], $C = aQ^b$ a	b
-	1.00	1.00	1.00	1.00	1.00	1.00	Raw Waste Characteristics	-	-
1	.38	.57	.38	.18	0	.30	Primary	.17	.62
2	.52	.78	.54	.82	0	.41	Primary and Metal Salt Addition ($FeCl_3$)	.23	.71
3	.79	.74	.78	.27	.10	.70	Primary and Trickling Filter	.24	.67
4	.88	.87	.88	.82	.10	.63	Primary and Trickling Filter with Metal Salt ($FeCl_3$)	.31	.75
5	.90	.91	.89	.36	.15	.74	Primary and Activated Sludge	.26	.69
6	.93	.93	.91	.82	.15	.75	Primary, Activated Sludge, Metal Salt (Alum)	.30	.72
7	.95	.91	.91	.27	.90	.94	Primary, Activated Sludge/Nitrification	.28	.67
8	.95	.91	.89	.27	.95	.95	Primary, Activated Sludge, Nitrification Denitrification	.40	.71
9	.95	.96	.89	.91	.90	.94	Primary Metal Salt Addition (Alum), Activated Sludge/Nitrification, Filtration	.36	.71
10	.98	.98	.94	.91	0	.76	Preliminary, Two-Stage Lime, Filtration, Carbon Adsorption	.80	.74
11	.98	.98	.92	.95	.95	.97	Primary Metal Salt Addition (Alum), Activated Sludge, Nitrification, Denitrification, Polymer, Filtration	.54	.72
12	.99	.98	.98	.95	.95	.97	Primary Metal Salt Addition (Alum), Activated Sludge, Nitrification, Denitrification, Polymer, Filtration, Carbon Adsorption	.70	.71

Note: Treatment systems include disinfection, sludge handling, miscellaneous structures, and support personnel.

[1] Source: EPA, Areawide Assessment Procedures Manual. App. H., EPA-600/9-76-014, Cincinnati, OH, 1976.

[2] UOD = Ultimate Oxygen Demand = $(1.5 \times BOD_5) + (4.57 \times NH_3\text{-}N)$.

[3] Treatment costs are annual costs in millions of 1976 dollars.

$$\begin{aligned} &\text{minimize } Z = \Sigma c_i\ x_i \\ &\text{subject to } \Sigma a_{ij}\ x_i \geqq \hat{y}_j \\ &\qquad\qquad\quad x_i = 0,1 \end{aligned} \tag{5-6}$$

where Z = total annual cost, \$/yr.

c_i = cost per unit of x_i,

a_{ij} = proportion of pollutant j removed per unit of x_i, and

$\hat{y}_j$ = water quality standard for pollutant j.

Because of the simple nature of the production functions, the solution to this problem is straightforward as illustrated by the following two examples.

Example 1. A single pollutant is to be controlled. Q = 10mgd. BOD_5 removal of at least 0.70.

Solution:

1) Find subset, S, of control systems such that $y_1 \geqq 0.70$

Answer. $S = \{3,4,\ldots, 12\}$

For systems 3 through 12, find the least cost solution.

Answer. Use system 3

$C_3 = 0.24(10)^{.67} = \$1.12 \times 10^6/\text{yr}.$

Example 2. Multiple pollutants to be controlled

$Q = 10$ mgd; $y_1 \geqq 0.70$, $y_2 \geqq 0.85$, $y_7 \geqq 0.90$

Solution:

1) Find intersecton of subsets which simultaneously satisfies $y_1 \geqq 0.70$, $y_2 \geqq 0.85$, and $y_7 \geqq 0.90$

Answer. $S_1 = \{3,4,\ldots, 12\}$

$S_2 = \{4,5,\ldots, 12\}$

$S_7 = \{7,8,9,11,12\}$

$S_1 \cap S_2 \cap S_7 = \{7,8,9,11,12\}$

For systems 7, 8, 9, 11, 12, find least costly option.

Answer. Select system 8.

$$C_8 = 0.28\ (10)^{0.67} = \$1.31 \times 10^6/\text{yr}.$$

For the simple case of a single plant, the economic analysis is straightforward. Find those systems which satisfy the standards. Then pick the least costly of those options. Example 2 will be used later to examine the related question of what portion of the total cost is attributable to removing 85% of the total suspended solids i.e., $y_2 \geq 0.85$? The answer to this question requires additional theory to be presented in the next chapter on cost allocation.

Single Plant With Variable Standard

Effluent standards can incorporate economic aspects by evaluating the total cost of meeting varying assumed required levels of control. Referring to Table 5-1, derive a cost curve as a function of % BOD_5 removal for a 10 mgd (37850 m^3/day) plant. The relevant data are presented in Table 5-2.

System	% BOD_5 Removal[a]	Annual Cost $\$ \times 10^6$	Remarks
1	38.0	0.71	
2	52.4	1.18	Inefficient; dominated by system 3
3	78.6	1.12	
4	88.1	1.74	Inefficient; dominated by system 5
5	90.5	1.27	
6	92.9	1.57	Inefficient; dominated by system 7
7	95.2	1.31	
8	95.2	2.05	Inefficient; dominated by system 7
9	95.2	1.85	Inefficient; dominated by system 7
10	97.6	4.40	Inefficient; dominated by system 11
11	97.6	2.83	
12	97.6	3.59	Inefficient; dominated by system 11

[a] Influent BOD_5 concentration = 210 mg/l.

Table 5-2. Annual Cost for a 10 mgd plant to remove BOD_5

Of the twelve control systems, seven are inefficient, i.e., they are dominated by another system such that

a) the same level of control can be provided at a lower cost; and/or

b) a higher level of control can be attained at an equal or lower cost.

The subset of five efficient control systems {1, 3, 5, 7, 11} are plotted in Figure 5-2. Incremental costs decrease between zero and 38%; remain constant to about 95%, and then rise sharply between 95.2 and 97.6% control. Incremental costs, as a function of % pollution control, and pounds of BOD_5 removed, are shown below:

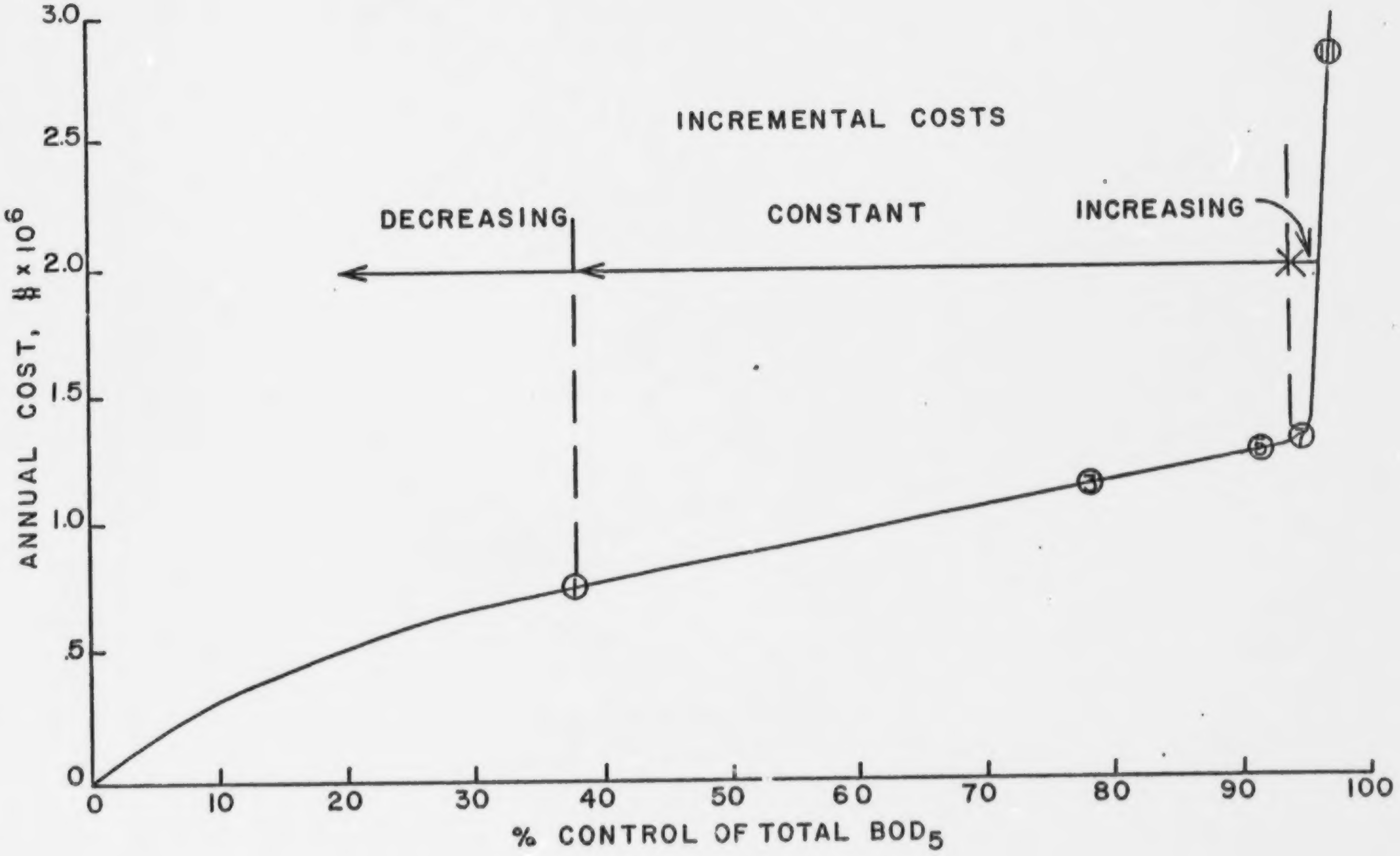

Figure 5-2. Annual Cost as a Function of % Control of BOD_5.

	Incremental Costs	
Range of BOD_5 Removal	$/%$BOD_5$ Removed	$/lb. BOD_5 Removed
0 - 38%	$18,700	$0.30
38 - 95.2%	$10,500	$0.17
95.2 - 97.6%	$633,000	$10.16

Table 5-3. Incremental Costs of BOD_5 removal

Figure 5-2 and Table 5-3 illustrate dramatically why there is a sound basis for encouraging secondary treatment of sewage. Incremental costs are relatively modest until about 95% BOD removal. However, pursuing control beyond that point becomes quite expensive, e.g., it costs more to increase control levels from 95.2% to 97.6% than it does to remove the initial 95.2%. Is this additional expenditure justifiable? If this information is provided to a decision maker, he may well decide that a "reasonable" standard would be to require 95% BOD_5 control - this point is popularly called the knee of the cost curve. The knee may be found by constructing a divided difference table and finding the point of maximum curvature. Alternatively, this point is located where the radius of a circle inscribing the arc at that point is a minimum.

REGIONALIZATION

Introduction

In the early years of federal participation in helping to finance the cost of sewage treatment plants, the analysis presented in the previous section would suffice. Not long after implementation of this policy, criticism was leveled that this approach was not cost effective in that opportunities for regionalization were not examined. Significant savings may be achieved by taking advantage of a) economies of scale, b) fuller use of assimilative capacity, and/or c) use of excess capacity. Following on the work of Deininger (25), a plethora of regional wastewater treatment optimization models were developed which showed that significant savings could be realized due to regionalization (see Loucks (26) for a summary of these models). In order to apply these approaches, the user needed to be familiar with relatively advanced concepts in optimization analysis. The regional problem was complicated by two factors: 1) the need to convert the Streeter-Phelps equation into an equivalent linear form; and 2) the need to represent the economies of scale in treatment correctly in the model. While the linearization of the Streeter-Phelps equation was straightforward, special algorithms were needed to handle economies of scale. This problem persists to date.

The beauty of the optimization models, once they are working properly, is that the solution so obtained is guaranteed to be the global optimum, i.e., it is uniquely the best solution. Operations research textbooks stress the dangers of suboptimizing wherein the regional problem is partitioned into subsets, each of which is then optimized with the hope (but no assurance) that the composite solution will be the global optimum. Experience has shown that such fragmented approaches yield solutions which are quite good, i.e., very close to the optimal solution. Also, very few of the regional optimal solutions have been implemented due to complications in financing and associated institutional arrangements.

Recognizing these problems, Smith et al. suggested the break-even pipe length approach for determining the economically feasible combinations (27). He considered two cases: (1) the economies of scale in treatment due to consolidation; and (2) the use of available excess capacity. His approach, which is limited to two sources, consists of determining the savings in treatment costs due to consolidation and using these savings to determine the maximum distance the two sources can be apart and still have the joint treatment be economically feasible. Smith suggested the use of break-even pipe length tables for determining the economically feasible combinations. In the next section, Smith's approach is generalized so that it is applicable to various cases that may arise in area-wide domestic wastewater management (28).

Optimization Procedure

Consider the case involving two sources, i and j. Assume that the two alternative domestic wastewater management strategies are: (1) each source treats its wastewater on-site; or (2) a joint treatment facility is located at downstream source j. The annual costs for dry-weather quality control can then be determined as follows:

Let $\bar{Q}_i$ = design capacity of the treatment facility required to serve source i, mgd;

Q_i = average annual wastewater flow originating at source i; and

L_{ij} = pipeline distance between source i and source j, miles.

ψ = level of treatment required for a single facility;

ϕ = level of treatment required for a joint facility;

C_i^{ψ} = annual costs for treatment facility located at source i, dollars per year;

C_{ij}^{ϕ} = annual costs for joint facility located at source j, dollars per year;

CA_i^{ψ} = amortized capital costs for treatment facility located at source i; and

OM_i^{ψ} = operation and maintenance costs for treatment facility located at source i, dollars per year.

The annual costs for dry-weather quality control

= annual treatment costs + annual transmission costs.

The annual treatment costs are comprised of amortized capital costs plus operation and maintenance costs.

Let TC^ψ = total treatment costs associated with treatment level, ψ, dollars per year;

CA^ψ = amortized capital costs associated with treatment level, ψ, dollars per year;

OM^ψ = operation and maintenance costs associated with treatment level, ψ, dollars per year; and

ψ = 1 (primary), 2 (secondary), or 3 (tertiary).

Then,

$$TC^\psi = CA^\psi + OM^\psi \tag{5-7}$$

According to the cost functions presented in Table 5-4,

$$CA^\psi = l_\psi \bar{Q}^{m_\psi} \tag{5-8}$$

$$OM^\psi = p_\psi Q^{q_\psi} \tag{5-9}$$

where l and p are the coefficients and m and q are the exponents in the cost functions for treatment level, ψ.

The total annual cost of wastewater transmission (dollars per year) as listed in Table 5-4 is

$$= 11{,}900\ Q^{0.51} L \tag{5-10}$$

Thus, the total treatment plus pipeline cost, C, is

$$C = CA^\psi + OM^\psi + 11{,}900\ Q^{0.51} L \tag{5-11}$$

A typical planning area usually involves many wastewater sources and treatment plant sites. Each source may have the option of treating its wastewater onsite or piping it to another location for treatment. From a regional standpoint, it is necessary to determine the annual costs associated with various strategies.

For on-site treatment, wastewater transmission costs are zero. In accordance with equation (5-7)

$$C_i^\psi = CA_i^\psi + OM_i^\psi = l_\psi \bar{Q}_i^{m_\psi} + p_\psi Q_i^{q_\psi} \tag{5-12}$$

Table 5-4.

Cost Functions for Domestic Wastewater Control[a,b,e,]

Treatment Level ψ	Type of Treatment	Annual Cost: \$/yr					
		Amortized Capital $CA = lQ^m$		Operation and Maintenance $OM = pQ^q$		Total $TC = uQ^v$	
		l_ψ	m_ψ	p_ψ	q_ψ	u_ψ	v_ψ
1	Primary	61,000	0.70	26,000	0.85	87,000	0.76
2	Primary + Secondary	118,000	0.77	55,000	0.83	173,000	0.79
3	Primary + Secondary + Tertiary	172,000	0.74	88,000	0.85	260,000	0.79

Total Costs for Wastewater Transmission, \$/yr[d] = $11{,}900\ Q^{0.51}$

Q = Dry-Weather Sewage Flow in million gallons per day (mgd)

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[a] Derived from data developed by Battelle-Northwest (44)

[b] Based on ENR* = 2200

[c] Based on 8 percent interest over 20 years.

[d] Cost per mile. Based on data by Smith (42).

[e] $Q \leq 100$ mgd. No economies of scale beyond 100 mgd.

*Engineering News Record.

and

$$C_j^{\phi} = CA_j^{\phi} + OM_j^{\phi} = l_{\phi}\bar{Q}_j^{m_{\phi}} + p_{\phi}Q_j^{q_{\phi}} \qquad (5\text{-}13)$$

The annual costs associated with the on-site control strategy are $C_i^{\psi} + C_j^{\psi}$. The annual costs associated with a joint treatment strategy are

$$C_{ij}^{\phi} = l_{\phi}(\bar{Q}_i + \bar{Q}_j)^{m_{\phi}} + p_{\phi}\,(Q_i + Q_j)^{q_{\phi}} + 11{,}900\; Q_i^{0.51}\, L_{ij} \qquad (5\text{-}14)$$

Thus, pipeline costs cannot exceed the savings from joint treatment, i.e.,

$$\text{savings} = C_i^{\psi} + C_j^{\psi} - C_{ij}^{\phi} \qquad (5\text{-}15)$$

Thus, the break-even pipe length L^*_{ij}, is

$$L^*_{ij} = \frac{\text{Savings in Treatment}}{11{,}900\; Q_i^{0.51}} \qquad (5\text{-}16)$$

Given the wastewater loads and the required level of treatment, the economic feasibility of joint treatment can be determined by comparing the actual pipeline distance between the two sources with the maximum pipeline distance that these sources can be apart. Application of equation 5-16 to various cases that arise in area-wide wastewater planning is presented below:

Case 1: Pipe effluent to site j to reduce treatment level at site i from ψ to ψ'

$$L^*_{ij} = \frac{C_i^{\psi} - C_i^{\psi'}}{11{,}900\; Q_i^{0.51}} \qquad (5\text{-}17)$$

Case 2: Pipe wastewater from i to j. No plant exists at present. L^*_{ij} can be found from equation (5-16).

Case 3: Pipe wastewater from i to j. Existing plant at j has excess capacity. City i pays incremental operation and maintenance costs. In this case,

$$L^*_{ij} = \frac{C_i^{\psi} - [p_{\phi}(Q_i + Q_j)^{q_{\phi}} - p_{\phi}(Q_j)^{q_{\phi}}]}{11{,}900Q_i^{0.51}} \qquad (5\text{-}18)$$

The above results can be extrapolated to more than two sites by simply redefining the ij pair as a single source and comparing the new source with another site. Note that an ij pair may be infeasible by itself, but becomes feasible when combined with another site k. This procedure lacks complete generality in that the partitioning of the entire set of sites into pairs is not unique. The resulting combinatorial problems make overall optimization unwieldy. Fortunately, the analyst's knowledge about the area and engineering judgment should permit him to derive a good solution based on this simple procedure.

Examples

The three cities shown in Figure 5-3 are considering a cooperative venture to reduce pollution control costs. Using cost data from Table 5-4, find the least costly way to provide:

- (a) tertiary treatment at city 1 acting alone. How far downstream could he afford to pipe his waste if he could discharge downstream after only primary treatment? Assume $\bar{Q} = Q$ for all cities.
- (b) secondary treatment for any two or three cities. Assume $\bar{Q} = Q$ for all cities.
- (c) secondary treatment for cities 2 and 3 given that city 3 has already built a secondary plant with $\bar{Q}$ = 5 mgd (18925 m^3/day) and Q = 2 mgd (7570 m^3/day). What is the breakeven pipe length for this case?

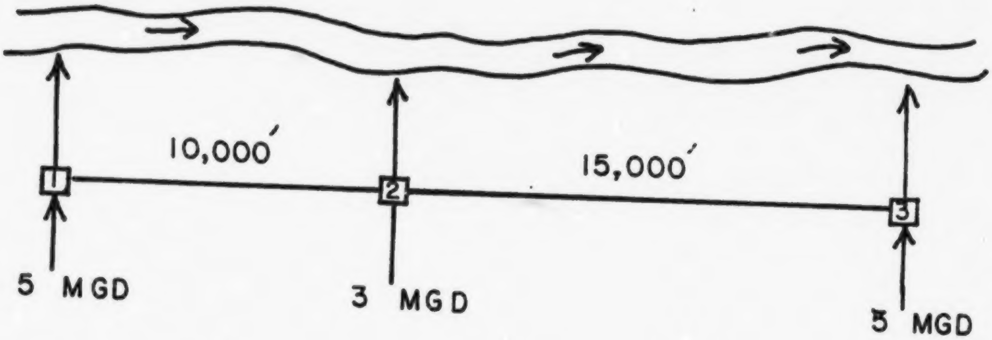

Figure 5-3. Configuration of Three Cities

Solution to Part (a) Case 1 Example

Cost of tertiary treatment at City 1

$$C_T = 260,000\ (5)^{.79} = \$927,000$$

Cost of primary treatment at City 2

$$C_P = 87,000\ (5)^{.79} = \$310,000$$

Savings in treatment = $C_T - C_P$ = \$617,000

$$\text{Maximum pipe length, } L^* = \frac{617,000}{11,900\ (5)^{.51}}$$

$$L^* = 22.8 \text{ miles } (36.7 \text{ km})$$

Solution to Part (b) Case 2 Example

1. Cost for three separate plants

$$C = 173{,}000Q^{0.79}$$

$$Q_1 = Q_3 = 5 \text{ mgd}, \; Q_2 = 3 \text{ mgd}$$

Substituting into the above equation yields

$$C(1) = \$617{,}000 \quad C(2) = \$412{,}000 \quad C(3) = \$617{,}000$$

2. Least cost solution for any two cities

a) 1 and 2

$$C(12)^* = \text{Min} \begin{cases} C(1) + C(2) & = \$1{,}029{,}000 \\ 173{,}000\ (8)^{.79} + 11{,}900\ (5)^{0.51}\ (1.89) & = \$\ \ 945{,}000 \end{cases}$$

C(12) = $945,000 (build one plant at 2)

b) 1 and 3

$$C(13)^* = \text{Min} \begin{cases} C(1) + C(3) & = \$1{,}234{,}000 \\ 173{,}000\ (10)^{.79} + 11{,}900\ (5)^{.79}\ (4.73) & = \$1{,}267{,}000 \end{cases}$$

C(13) = $1,234,000 (build two plants)

c) 2 and 3

$$C(23)^* = \text{Min} \begin{cases} C(2) + C(3) & = \$1{,}029{,}000 \\ 173{,}000\ (8)^{.79} + 11{,}900\ (3)^{.79}(2.84) & = \$\ \ 975{,}000 \end{cases}$$

C(23) = $975,000 (build one plant at 3)

d) 1, 2, and 3

$$C(123) = \text{Min} \begin{cases} C(1) + C(2) + C(3) & = \$1{,}646{,}000 \\ C(12) + C(3) & = \$1{,}562{,}000 \\ C(13) + C(2) & = \$1{,}646{,}000 \\ C(23) + C(1) & = \$1{,}592{,}000 \\ 173{,}000\ (13)^{0.79} + 11{,}900\ (5)^{.52}(1.89) & \\ \quad + 11{,}900\ (8)^{.51}(2.84) & = \$1{,}461{,}000 \end{cases}$$

C(123)* = $1,461,000 (build one plant at 3)

Solution to Part C - Case 3 Example

Cost of adding City 2 to City 3 given that City 3 already has plant is incremental operation and maintenance cost only. Using Equation 5-18

$$L_{23}^* = \frac{173{,}000\ (3)^{.79} - [55{,}000\ (5)^{.83} - 55{,}000\ (2)^{.83}]}{11{,}900\ (3)^{.51}}$$

$L_{23}^* = 14.43$ miles (23.2 km)

GENERALIZED COST-EFFECTIVENESS ANALYSIS

In the previous cases, the economic analysis was relatively straightforward because the production functions relating resource inputs and pollution control as the output were very simple. However, the analysis required at present includes not only alternative treatment plant configurations but also equalizing storage-treatment systems for wastewater control and wet-weather control, and best management practices such as street sweeping, sewer repair and/or flushing, etc. In addition, opportunities exist for multi-purpose schemes, e.g., storage for drainage control and pollution control. The selected production function plays a vital role in the analysis. It represents a summary of the scientific and technical knowledge about the problem. For our purposes, the most complicated production function will be of the following form

$$y = f(x_1, x_2)$$

where y = proportion of pollutant removed,

x_1 = amount of input x_1, and

x_2 = amount of input x_2.

The cost functions will be additive and either

a) linear $Z = \Sigma c_i\ x_i$, or

b) nonlinear $Z = \Sigma c_i\ x_i^{\beta_i}$.

Graphical and analytical solutions for these two cases are presented below.

Linear Cost Functions

If the cost function is linear then the problem may be formulated as follows:

$$\begin{aligned} &\text{minimize } Z = c_1\ x_1 + c_2\ x_2 \\ &\text{subject to } y = f(x_1\ , x_2) \\ &\qquad x_1, x_2 \geq 0 \end{aligned} \tag{5-19}$$

The problem can be solved graphically as shown in Figure 5-4.

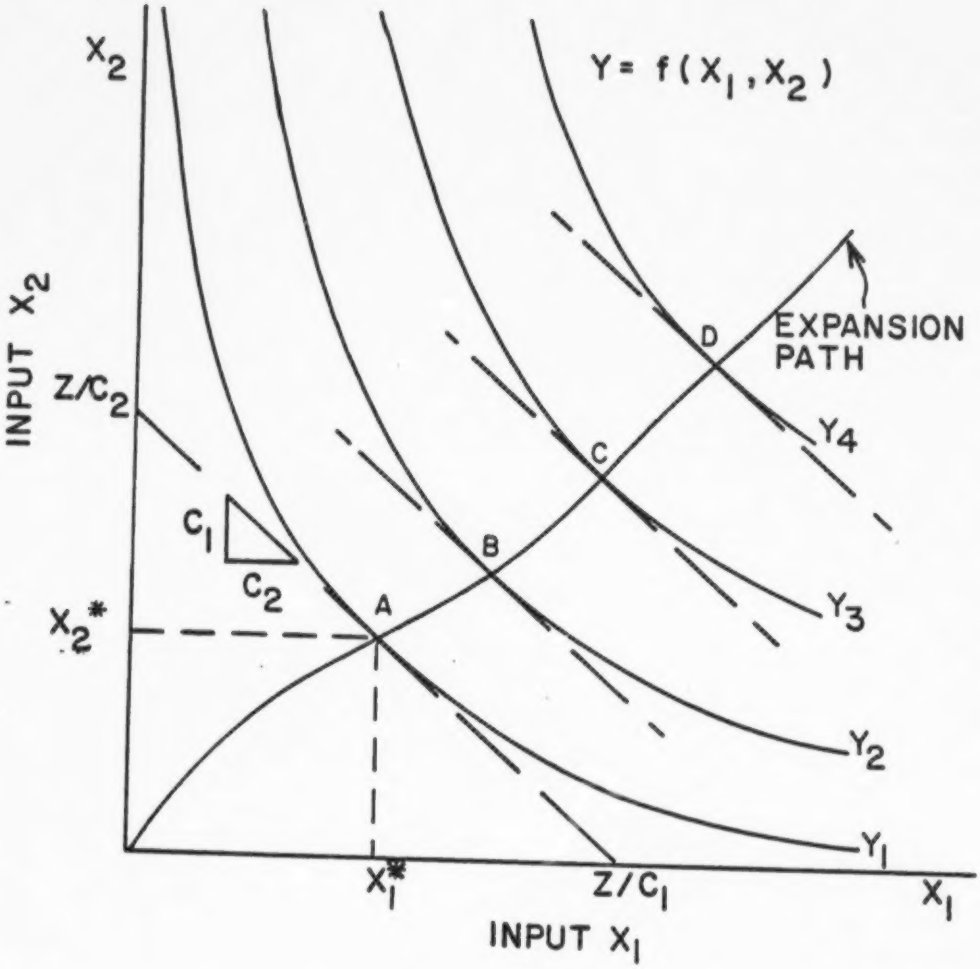
Y = f(X₁, X₂)
X₂
INPUT X₂
Z/C₂
C₁
C₂
X₂*
A
B
C
D
EXPANSION PATH
Y₄
Y₃
Y₂
Y₁
X₁*
Z/C₁
X₁
INPUT X₁

Figure 5-4. Graphical Solution to Two Input, One Output Problem.

For a given y, say y_1, the problem is to find the least costly combination of x_1 and x_2 for producing y_1. The graphical solution can be done as follows:

a) Assume an available budget, Z (A good trial value of Z can be obtained by picking intermediate values of x_1 and x_2 and substituting into $Z = c_1 x_1 + c_2 x_2$)

Using this trial value of Z, solve for x_2, i.e.,

$$x_2 = Z/c_2 - c_1/c_2(x_1) \tag{5-20}$$

and plot this equation onto Figure 5-4. Then, move parallel to this line until it is tangent to the isoquant, $y_1 = f(x_1,x_2)$ as shown in Figure 5-4. Note that if $x_1 = 0$, then $x_2 = Z/c_2$, and if $x_2 = 0$, then $x_1 = Z/c_1$. The initially selected value of Z was arbitrary. It is the relative unit costs, c_1/c_2 which are important. This process is repeated for various assumed values of y until the expansion path is defined. The final result can then be presented as shown in Figure 5-5.

Nonlinear Cost Functions

The analysis becomes more complicated if the cost function is additive but nonlinear, e.g.,

$$Z = c_1 x_1^{\beta_1} + c_2 x_2^{\beta_2} \tag{5-21}$$

The graphical solution procedure is as before. A trial value of Z is selected. Then the cost function is plotted. The resulting shapes of the cost function are shown in Figure 5-6 for the following example cost function

$$Z = 3x_1^{\beta_1} + 2x_2^{\beta_2}.$$

Let β_1 and β_2 assume the values 0.5, 1, 1.5, and 2. Figure 5-6 shows the shape of the cost function for various assumed values for the β's. If $\beta_1, \beta_2 \geq 1$, then the function is concave to the origin. In this case, the point of tangency with the production function satisfies necessary and sufficient conditions for optimality. However, if $\beta_1, \beta_2 < 1$, then the cost function is convex as is the production function so the point of tangency may not be optimal depending on the relative curvature of the cost and production functions. Thus, extra care must be exercised in analyzing such cases.

The other possibility is that the values may be both greater than and less than 1 as shown in Figure 5-7. Then the function is neither convex nor concave so it is necessary to analyze such cases with extreme care.

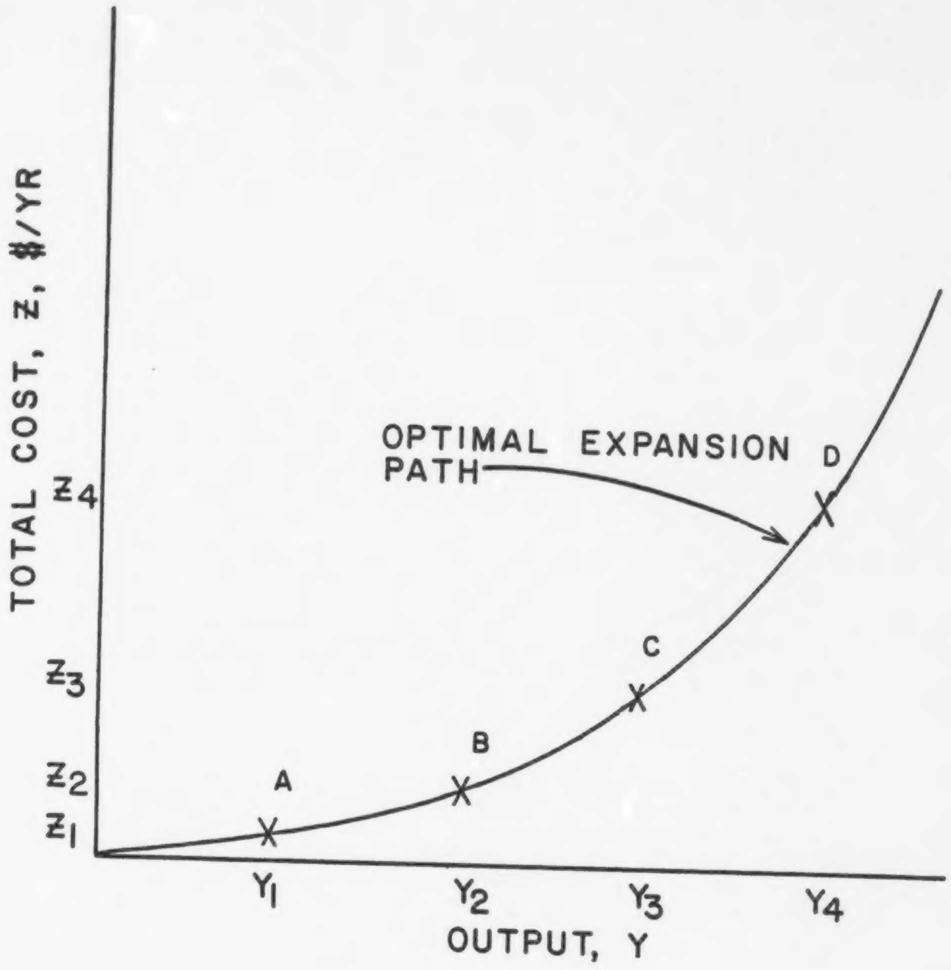

Figure 5-5. Optimal Expansion Path.

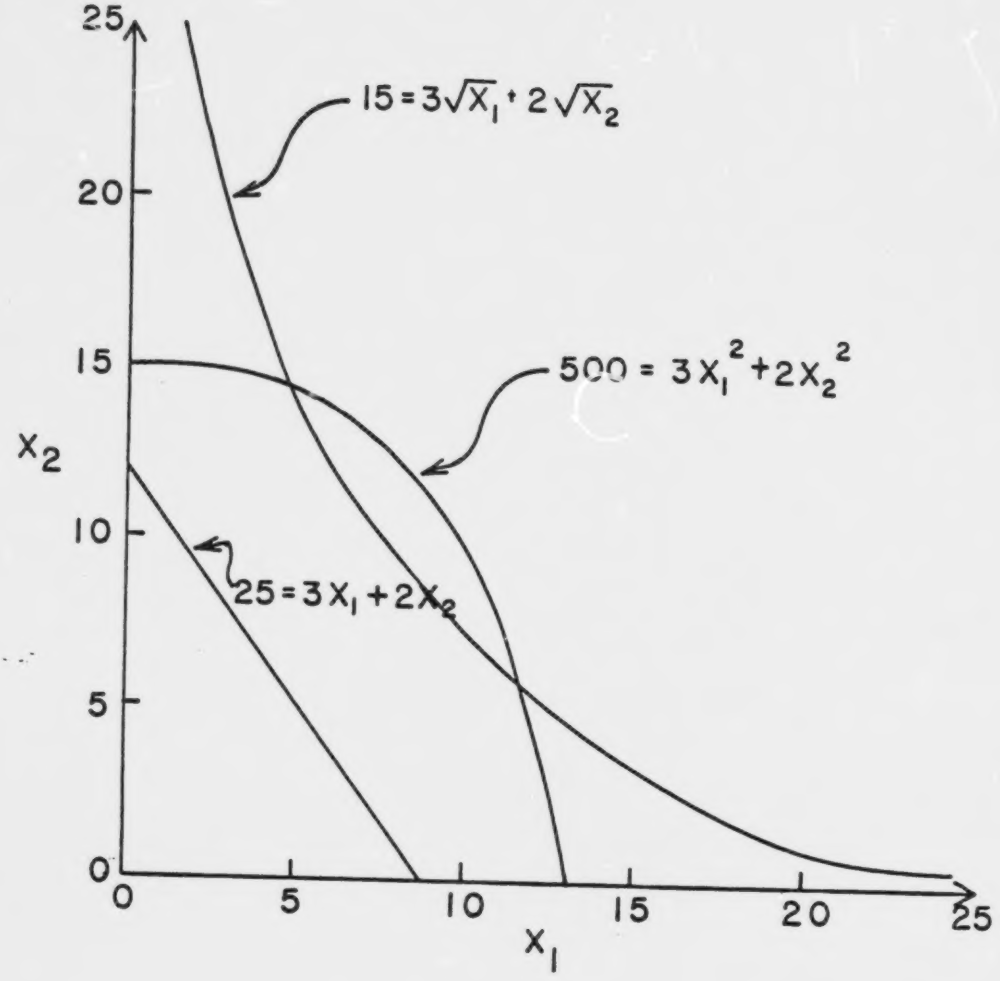

Figure 5-6. Shape of $Z=3X_1^{\beta_1} + 5X_2^{\beta_2}$ for Assumed Values of Z, β_1, & β_2.

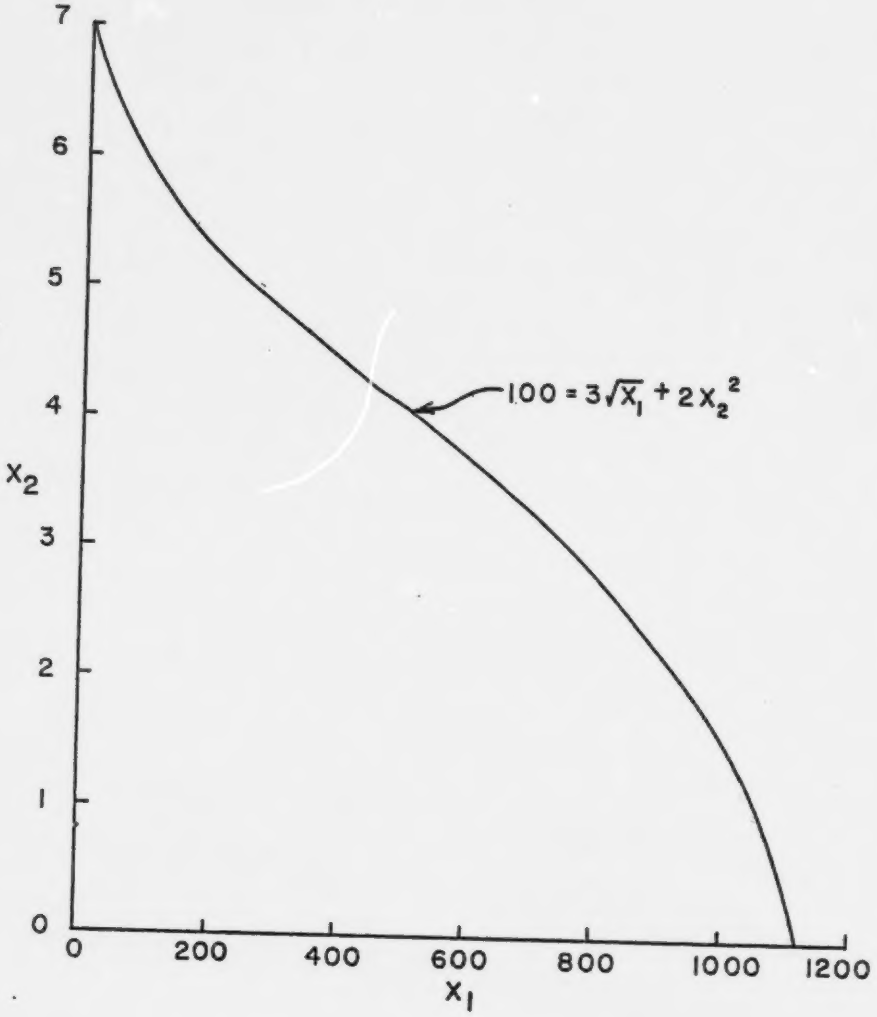

Figure 5-7. Shape of Cost Function With Exponents Greater Than and Less Than 1.

Applications to Equalizing Storage in Sewage Treatment

Recall the problem of evaluating the desirability of flow equalization to reduce peak flows in sewage treatment plants. The production function, shown in Figure 4-4 and repeated as Figure 5-8, indicates that it may be desirable to consider intermediate possibilities to the traditional two options of no or total flow equalization. To illustrate the application of the production function approach, assume the following annualized unit costs.

Item	Unit Cost
Storage	\$500,000/mg. (\$132.10/m^3)
Primary Treatment	50,000/mgd (\$13.21/m^3/day)
Secondary Treatment	100,000/mgd (\$26.42/m^3/day)
Tertiary Treatment	150,000/mgd (\$39.63/m^3/day)

The problem is to

$$\text{minimize } Z = c_S(S) + c_T(T)$$

subject to the production function (see Figure 5-8).

Assume Z = \$500,000. Then the equation of the budget line is

$$S = Z/c_S - c_T/c_S\ (T), \text{ or}$$

$$S = \frac{1.0 - c_T(T)}{500,000}$$

Budget lines for the three levels of treatment are plotted in Figure 5-8. The optimal solutions for the three cases are shown below:

Case	T^*, mgd (m^3/day)	S^*, mgd (m^3/day)
1) Storage + Primary Treatment	10.6 (40100)	0.14 (530)
2) Storage + Secondary Treatment	9.2 (34800)	0.36 (1360)
3) Storage + Tertiary Treatment	8.4 (31800)	0.60 (2270)

Note that storage is substituted for treatment as progressively more expensive treatment is used. In all cases, an intermediate mix of storage and treatment is optimal. For the secondary treatment case, the comparative costs are

Option	Cost, \$/yr
1) Secondary treatment only	\$1,344,000
2) Secondary treatment with complete flow equalization	\$1,428,000
3) Optimal mix of treatment and storage	\$1,100,000

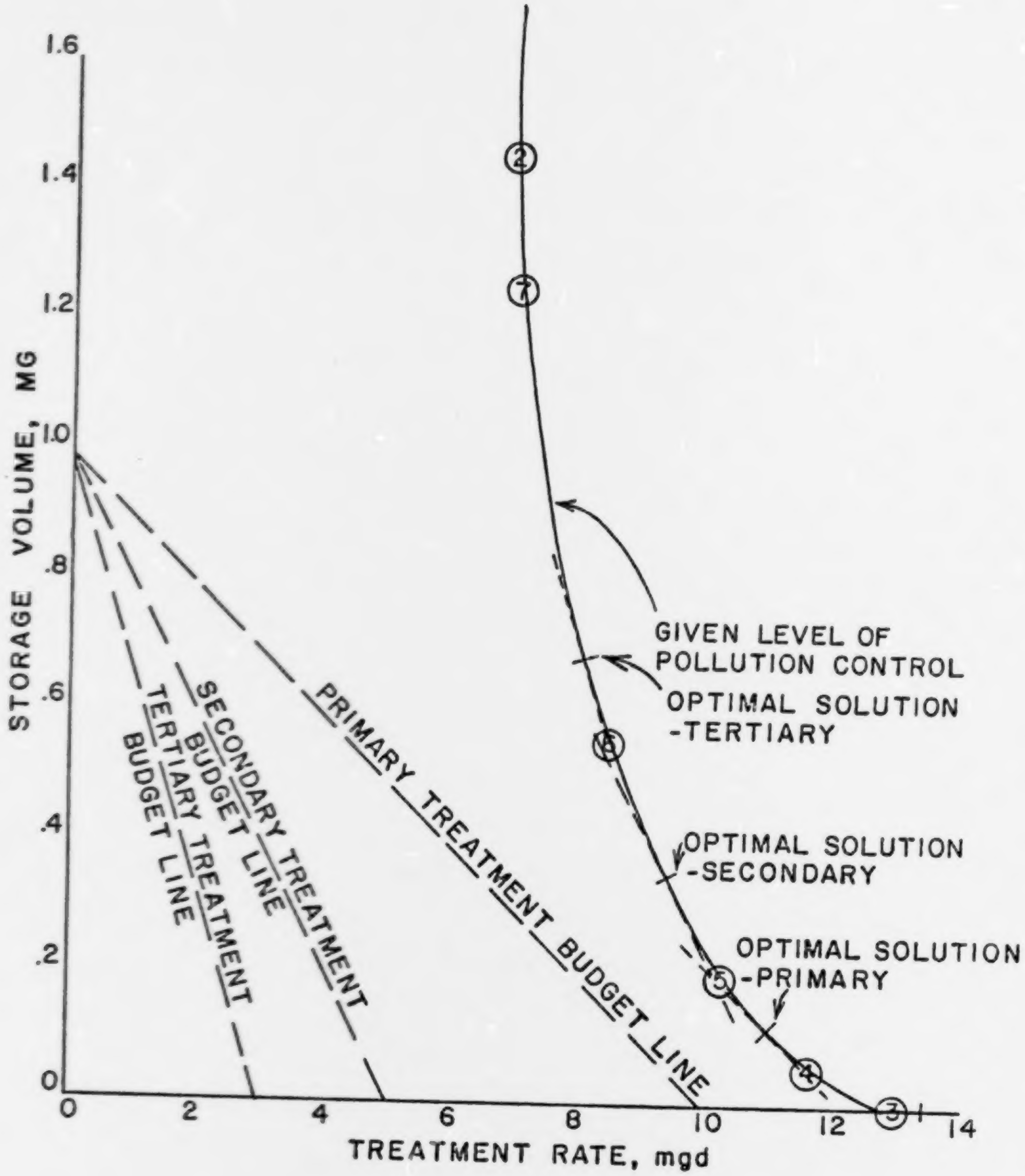

Figure 5-8. Production Function for Equalizing Storage in Sewage Treatment Plants.

In this case, the savings from using the optimal solution are substantial, 18 percent less than the second best solution.

Application to Storage-Treatment for Wet-Weather Control

As described in the previous chapter, the STORM model was used in an earlier study to generate isoquants of the percent pollutant control, y, as a function of treatment rate, x_1, and storage volume, x_2. Results for Minneapolis are shown in Figure 5-9.

...Graphical Optimization

The question of what functional form is suited to a particular optimization problem is avoided by a graphical solution procedure. The general optimization problem for storage-treatment is

$$\text{minimize } Z = c_1 x_1^{\beta_1} + c_2 x_2^{\beta_2} \qquad (5\text{-}22)$$

$$\text{subject to } g(y, x_1, x_2) = 0 \qquad (5\text{-}23)$$

$$x_1, x_2 \geqq 0$$

where Z = total annual control costs, \$/acre (\$/m^2),

β_1, β_2 = cost exponents,

c_1 = unit cost of x_1, annual \$/acre-in/hour (\$/m^3/hr), and

c_2 = unit cost of x_2, annual \$/acre-in (\$/m^3).

$g(y, x_1, x_2) = 0$ is the production function derived by interpolation between data points.

The optimal combination of x_1 and x_2 and the associated cost, Z*, for any value of y is found by locating the point of tangency of the isoquant (equation 5-23) and isocost (equation 5-22) lines giving the lowest value of Z.

...Analytical Optimization

The formulation of the optimization model using the exponential production function, i.e., equation 4-6 and assuming linear control costs, is (9):

$$\text{minimize } Z = c_1 x_1 + c_2 x_2 \qquad (5\text{-}24)$$

$$\text{subject to } x_1 = \underline{x}_1 + (\bar{x}_1 - \underline{x}_1)e^{-kx_2} \qquad (5\text{-}25)$$

$$x_1, x_2 \geqq 0$$

Solving this constrained optimization problem yields

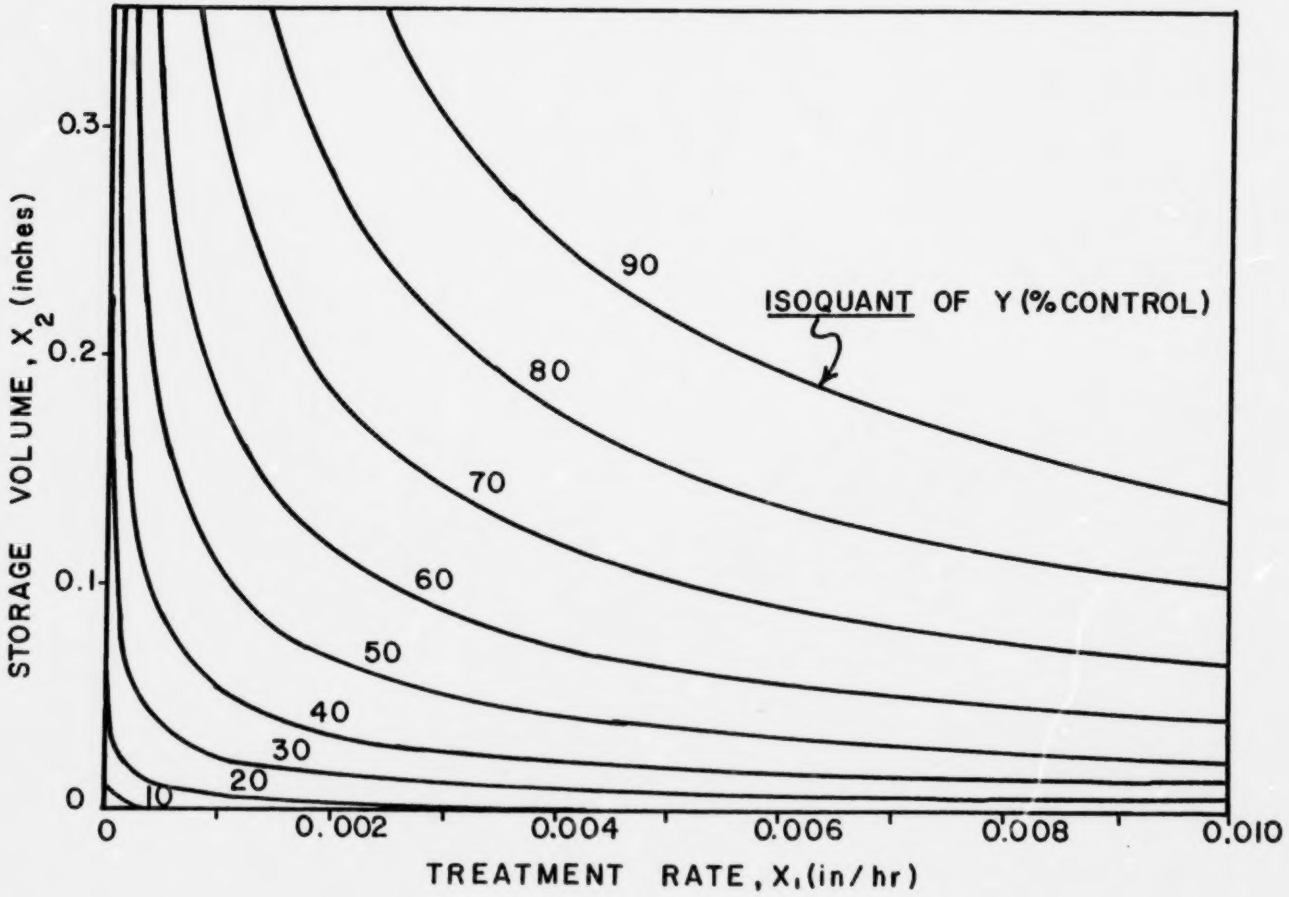

Figure 5-9. Storage-Treatment Isoquants for Minneapolis, Minnesota. (14).

$$x_2^* = \max \left[\frac{1}{k} \ln \frac{c_1}{c_2} [k(x_1 - \underline{x}_1), 0] \right] \tag{5-26}$$

where x_2^* = optimal amount of storage, inches,

$$\text{and } x_1^* = \underline{x}_1 + (\bar{x}_1 - \underline{x}_1)e^{-kx_2^*} \tag{5-27}$$

where x_1^* = optimal amount of treatment, inches/hour (cm/hr).

Note that x_1^* is a function of x_2^* so it is necessary to find x_2^* first. Knowing x_1^* and x_2^*, the optimal solution is

$$Z^* = c_1 x_1^* + c_2 x_2^* \tag{5-28}$$

where Z^* = total annual cost for optimal solution, \$/acre (\$/m^2).

The procedure used to find Z^* for any level of control was to solve the optimization problem for several assumed values of y and then fit a function to the result. The equation of best fit was:

$$Z^* = me^{\gamma} \tag{5-29}$$

where m, γ = coefficients.

This equation is unsuitable at low levels of y since $Z^* \to m$ as $y \to 0$.

The above problem is formulated below using a Cobb-Douglas production function and a nonlinear cost function with economies of scale in storage and treatment.

$$\text{Minimize } Z = c_1 x_1^{\beta_1} + c_2 x_2^{\beta_2} \tag{5-30}$$

$$\text{Subject to } y = A_1 x_1^{\alpha_1} x_2^{\alpha_2} \tag{5-31}$$

$$x_1, x_2 \geqq 0$$

where β_1, β_2 = cost exponents and $\beta_1 < 1$ implies economies of scale in the cost function.

Solution of this problem yields:

$$x_1^* = \left(\frac{\alpha_1}{\alpha_2} \frac{\beta_2}{\beta_1} \frac{c_2}{c_1} \right)^{\frac{1}{\beta_1}} x_2^{\frac{\beta_2}{\beta_1}} \tag{5-32}$$

Note that the optimal combination of x_1 and x_2 is independent of y. The final solution is shown below:

$$Z^* = Ky^{\frac{\beta_1 \beta_2}{\alpha_1 \beta_2 + \alpha_2 \beta_1}} \tag{5-33}$$

$$\text{where } K = c_2 \left(\frac{\beta_2\alpha_1 + \beta_1\alpha_2}{\beta_1\alpha_2} \right) \left[\frac{\left(\frac{\beta_1 c_1 \alpha_2}{\beta_2 c_2 \alpha_1} \right)^{\alpha_1}}{A^{\beta_1}} \right]^{\frac{\beta_2}{\alpha_2\beta_1 + \alpha_1\beta_2}} \tag{5-34}$$

Equations 5-32 and 5-33 are simplified if linear costs are assumed, i.e., $\beta_1 = \beta_2 = 1$.

The solution shown in equation 5-33 provides an excellent summary characterization of whether overall economies of scale exist. Most environmental control facilities exhibit economies of scale in the cost of construction and operation and maintenance, i.e., $\beta_i < 1$. However, these savings are offset by the diminishing marginal productivity of the inputs, i.e., $\Sigma\alpha_i < 1$. Thus, the general test for the effect of scale is whether

$$\frac{\beta_1 \beta_2}{\alpha_1\beta_2 + \alpha_2\beta_1} \quad \begin{aligned} &> 1 \rightarrow \text{diseconomies of scale} \\ &= 1 \rightarrow \text{constant returns to scale} \\ &< 1 \rightarrow \text{economies of scale} \end{aligned}$$

Furthermore, it is relatively straightforward to do sensitivity analysis using equation 5-33. Of course, the Cobb-Douglas production function is not as accurate as equation 5-25. Nevertheless, it may be preferable to use it in certain cases.

...Example Comparison

With data for the developed portion of Minneapolis and linear costs reported earlier, an optimal strategy is derived using each of the three methods (23, 29). The developed area of Minneapolis encompasses 215,000 acres (87,100 hectares) and has a population density of 7.92 persons per acre (19.55 persons per hectare). The runoff for the year used to generate the data points with STORM was 10.50 inches (26.67 cm). The unit cost of secondary treatment, c_1, is \$9810/ac-in/hr (\$95.50/m^3/hr), whereas the unit cost of storage, c_2, is \$219/ac-in (\$2.13/m^3). Secondary treatment is assumed to have a constant removal efficiency of 85 percent. In all the above equations, y is assumed to be pollutant control at a removal efficiency of 100 percent. If another efficiency is desired, the following transformation is made:

$$y' = \eta y \tag{5-35}$$

where y' = actual percent pollutant control, $0 \leqq y' \leqq \eta$, and

η = removal efficiency, $0 \leqq \eta \leqq 1.00$.

Application of the graphical method and the expansion path produced by each method is shown in Figure 5-10. The cost equation and optimal strategy for various levels of pollutant (BOD) control are shown in Table 5-5. The results reveal discrepancies among the methods, but all are reasonably close to the answer provided by the graphical solution. The graphical solution is assumed to be the "best" answer since it is unaffected by the error introduced by fitting functions. However, for preliminary estimates, each method seems to provide results well within the accuracy required for such an analysis.

Application to BMP'S and Agricultural Production Modification

This section extends the seven step procedure (23) to permit evaluation of agricultural areas. The main difference between the urban and agricultural problem is that the farmer may consider production modifications in response to an imposed restriction on pollutant discharges. Thus, it is necessary to estimate the response that he might make if confronted with such a situation. The method used to measure this response is an expansion of a farm production model to include constraints on pollutant discharges. With this formulation, the allowable pollutant discharge is varied from a large amount down to zero. The farmer is assumed to respond to the restrictions as best he can in that he tries to minimize his losses. A linear programming model of a dairy farm, developed by Haith and Atkinson, is used to illustrate this procedure (31).

The other control option to be considered as an alternative to production modification is a detention basin which can be installed at the downstream end of the farm. The performance of this basin is estimated based on information presented by Chen et al. (32).

Knowing the farmer's production modification possibilities and the comparative effectiveness of the detention basin, the graphical evaluation procedure is applied to determine the optimal blend of these two options. The result is a single cost function which represents the overall optimal response to any level of pollutant discharge restrictions.

The seven step procedure can be applied in agricultural areas to any mix of control options in series, parallel or both. The difficult part of the analysis is to derive realistic production and cost functions. Due to the availability of a dairy farm production model, the methodology is applied to a 306 acre (124 hectare) (tillable area) dairy farm described in Haith and Atkinson (29). They present a linear programming model which allows determination of the production adjustments that could be made on a dairy farm facing pollution constraints. For this application, production adjustments and a detention basin excavated downstream on non-tillable land are the options available to the farmer. Sediment washoff is the pollutant discharge to be controlled since sediment loss is used as the indicator of nutrient loadings in surface runoff. For example, the NPS Model simulates all pollutants as a function of sediment washoff (33).

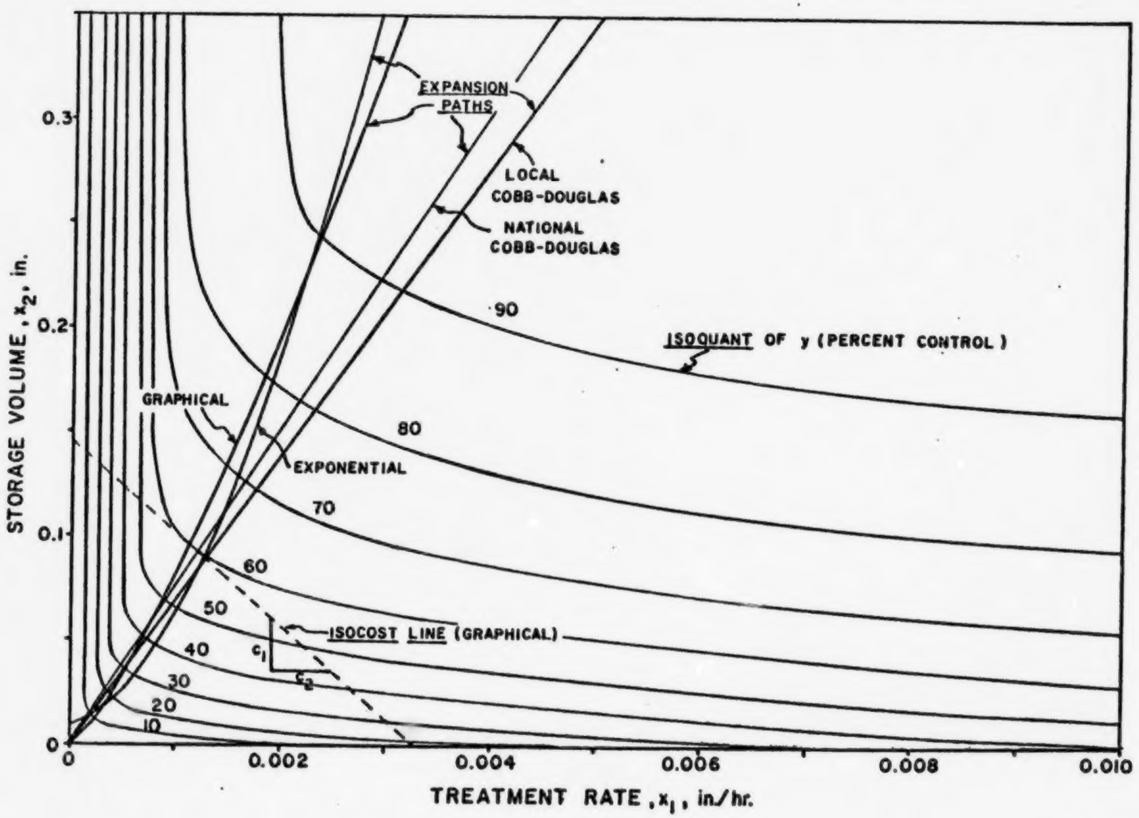

Figure 5-10. Application of Graphical Method and Expansion Paths.

TABLE 5-5. - Results of System Optimization, Minneapolis, Minnesota.

Percent Pollutant (BOD) Control, y', $0 \leq y' \leq 85$	Graphical Method			Exponential $Z^* = 4.65R^{0.0382y'}$			Local Cobb-Douglas $Z^* = 0.0331(y')^{1.832}$			National Cobb-Douglas $Z^* = 0.0907^*(y')^{1.605}$		
	x_1^{*a}	x_2^{*b}	Z^{*c}	x_1^{*a}	x_2^{*b}	Z^*	x_1^{*a}	x_2^{*b}	Z^{*c}	x_1^{*a}	x_2^{*b}	Z^{*c}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
10	0.00028	0.014	6	0.00027	0.017	7	0.00009	0.006	2	0.00014	0.011	4
25	0.00052	0.036	13	0.00058	0.032	13	0.00048	0.034	12	0.00061	0.046	16
50	0.0011	0.092	31	0.00125	0.092	32	0.00170	0.119	43	0.00184	0.138	48
75	0.0023	0.235	74	0.00238	0.254	79	0.00358	0.251	90	0.00354	0.266	93

[a] In inches per hour.

[b] In inches.

[c] In dollars per acre.

The Haith and Atkinson linear programming model is a relatively simple model which focuses on the nutrient budgets of a dairy farm. Farm income is maximized subject to constraints on land, manure handling, pollutant loss, and feasible soil-crop combinations. The soil loss constraint is as follows:

$$\sum_{i,j} s_{ij}X_{ij} \le b_i \qquad (5\text{-}36)$$

where s_{ij} = annual soil loss from soil type i in land use j as determined by the universal soil loss equation; X_{ij} = acres of soil type i in land use j; and b_i = maximum allowable loss from soil type i. The other pollution constraints, e.g., particulate phosphorus and dissolved nitrogen, are similar.

In order to determine the farmer's response to the pollutant constraint, it is necessary to know the marginal cost of pollutant control at any level (i.e., the $\partial Z/\partial b_i$ where b_i is the right hand side of equation 5-36 and Z is the net farm income). This information can be found by modifying the objective function and doing post-optimal analysis. The objective function is as follows:

$$\text{Maximize } Z = rH - \text{cost of farm operation} \qquad (5\text{-}37)$$

where r is the average rate of return per cow and H is the herd size. For this analysis, the rate of return per cow is expressed as the marginal rate of return rather than the average rate of return for a given herd size. Table 5-6 summarizes the difference between the average and the marginal rate of return. Substituting $\sum_k MR_k H_k$ for rH in equation 5-36 gives a piece-wise linear approximation for the total return for a given herd size where MR_k is the marginal rate of return in interval k and $\sum_k H_k$ is equal to the herd size.

Herd Size[a]	Average Return per Cow[a], r	Interval, k	Herd Size in Interval k, H_k	Approximate Marginal Return per cow, MR_k
0				
50	298	0-50	50	298
70	276	50-70	20	221
90	247	70-90	20	146
110	228	90-110	20	143

[a] From (31)

Table 5-6. Average and Marginal Rate of Return per Cow

The post-optimal analysis examines the effect on farm income of varying the maximum allowable soil loss per unit area from zero to the level necessary to maximize farm income. Profits with the herd size of 110 are \$11,776 and the soil loss is 0.57 tons/acre-year (1.28 metric tons/hectare-year); hence this discharge represents zero percent control. The sensitivity analysis varies the maximum allowable soil loss per unit area from 0.57

tons/acre-year (1.28 metric tons/hectare-year) to zero tons/acre-year (metric tons/hectare-year) (i.e., 100 percent control) and gives the resulting profits at desired increments. The total cost of controlling the pollutant is the difference between the maximum profits ($11,776) and the profit of the given level of control. The cost to the farmer is the profits foregone due to satisfying the soil loss constraint. The relationship between total costs and various levels of control using production adjustments is shown in Figure 5-11.

For the downstream option, the production function and cost function for the detention basin are needed. Chen et al. present detention basin capacity vs. sediment removal functions for construction site drainage areas (32). These construction sites generally resemble fallow ground conditions in agricultural practices. For this simplified application, the performance of the detention basin is assumed to be independent of sediment concentration since the performance must be in terms of the fraction removed due to the nature of the serial operation. For instance, if the basin were to be sized such that it would remove 50 percent of the sediment washoff resulting from fallow ground condition, the basin is also assumed to remove 50 percent of the sediment washoff resulting from lesser loadings. The relationship between basin capacity and the fraction of sediment washoff removed for the 306 acre (124 hectare) drainage area (i.e., the dairy farm's tillable area) is shown in Figure 5-12. Applying an annual unit cost of $25 per acre-inch ($\$.24/m^3$) of basin capacity gives the total cost curve, Figure 5-13, for sediment removal using this detention basin.

Since the total cost curve is known for each option acting alone, the isoquants of the fraction of sediment washoff controlled by the serial operation of the two options is found by minimizing the combined costs subject to the following equation:

$$Y = Y_M + Y_{DB} (1 - Y_M) \qquad (5\text{-}38)$$

where Y is the overall fraction removed, Y_M is the fraction controlled by production modification and Y_{DB} is the fraction removed by the detention basin. The optimal expansion path is the line through the points of tangency between the isoquants and the isocost lines, as shown in Figure 5-14.

The least costly solution for controlling 0.3 or less of the sediment wash-off is to use only the detention basin (i.e., the expansion path is along the Z_{DB} axis between the zero and the 0.3 level of control). Beyond 30 percent control, the optimal solution is a blend of the two options. The final results of the methodology, applied to the dairy farm example, are illustrated by Figure 5-15. This figure compares the total cost curve for the most efficient mix of the control options with the total cost curve for each option acting alone for any level of pollution discharge restriction. These results suggest that savings can be realized by the application of this procedure to agricultural areas faced with a variety of pollution control options.

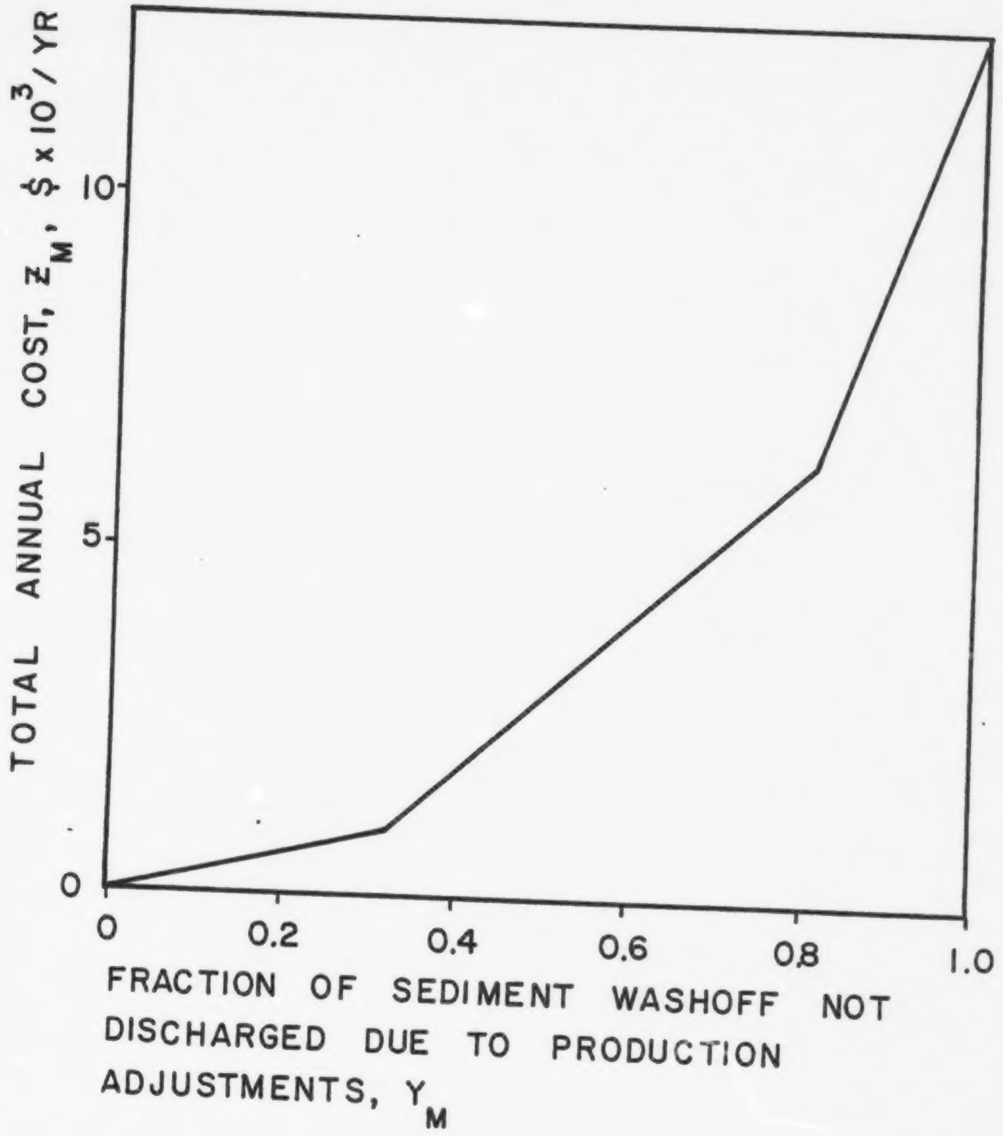

Figure 5-11. Total Cost Curve for Farm Production Modification.

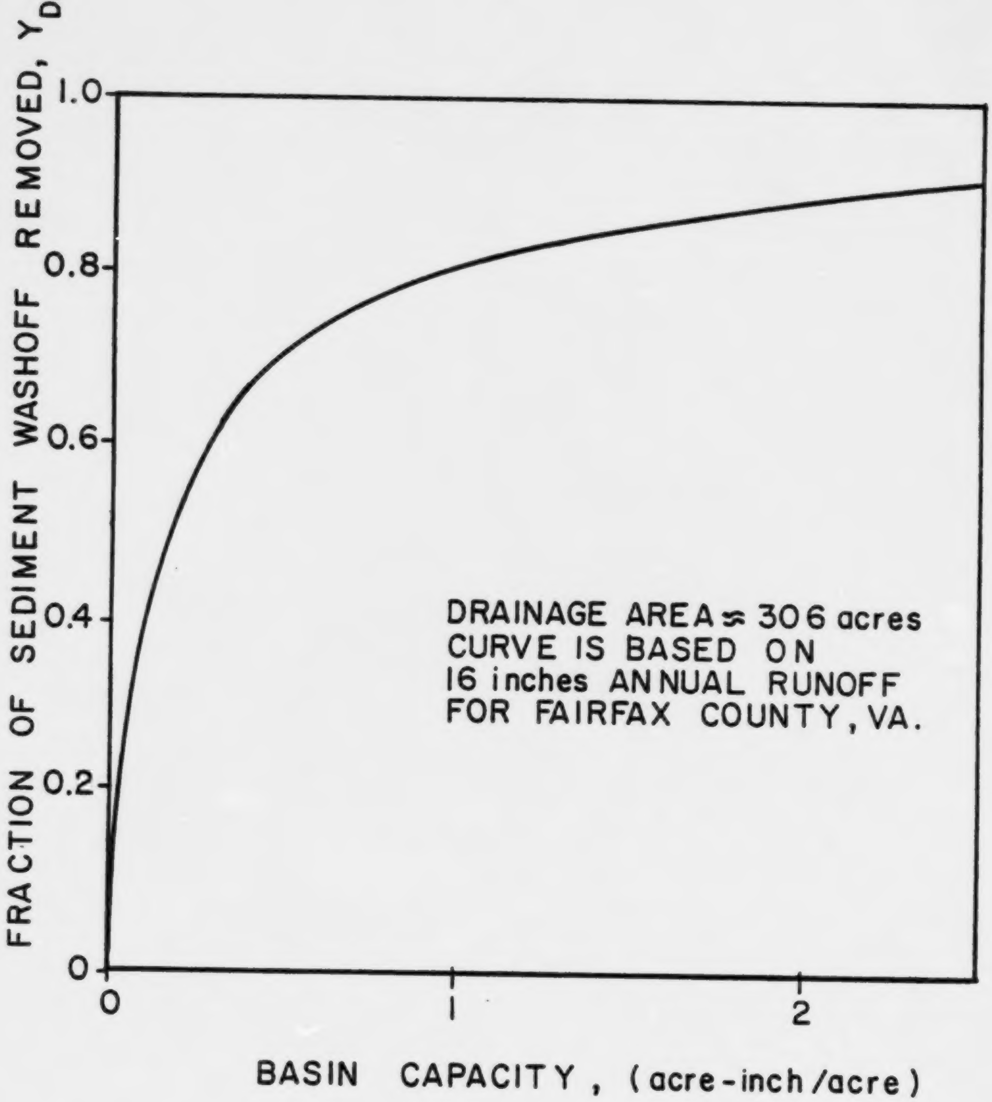

Figure 5-12. Detention Basin Production Function (52).

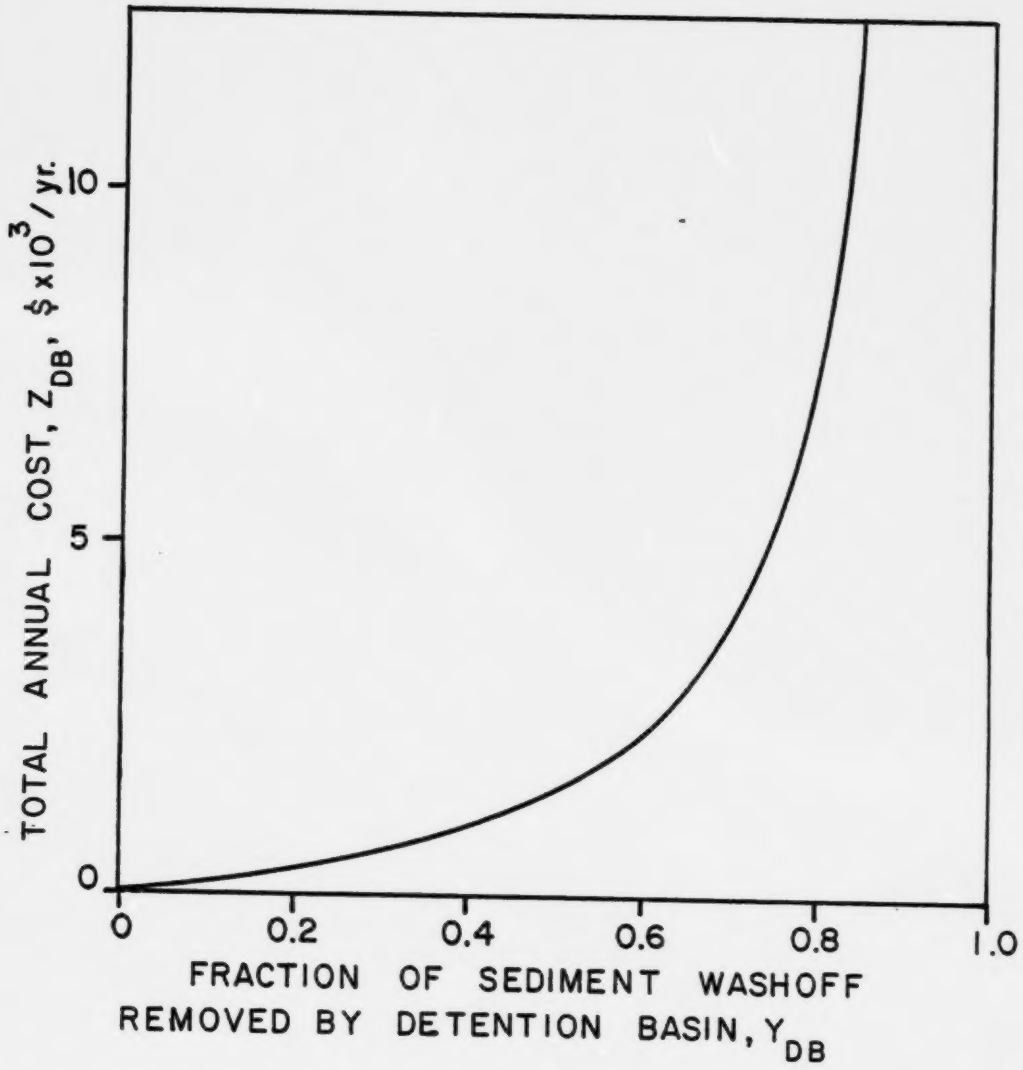

Figure 5-13. Total Cost Curve for Detention Basin.

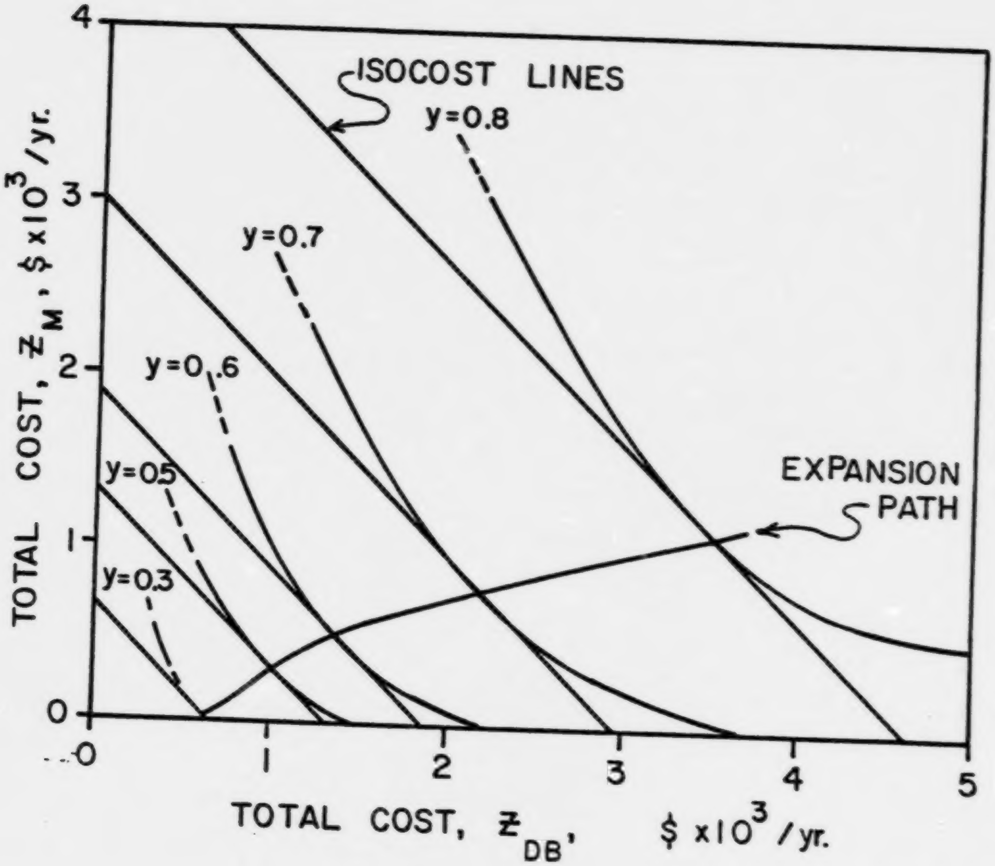

Figure 5-14. Optimal Expansion Path for Overall Sediment Control.

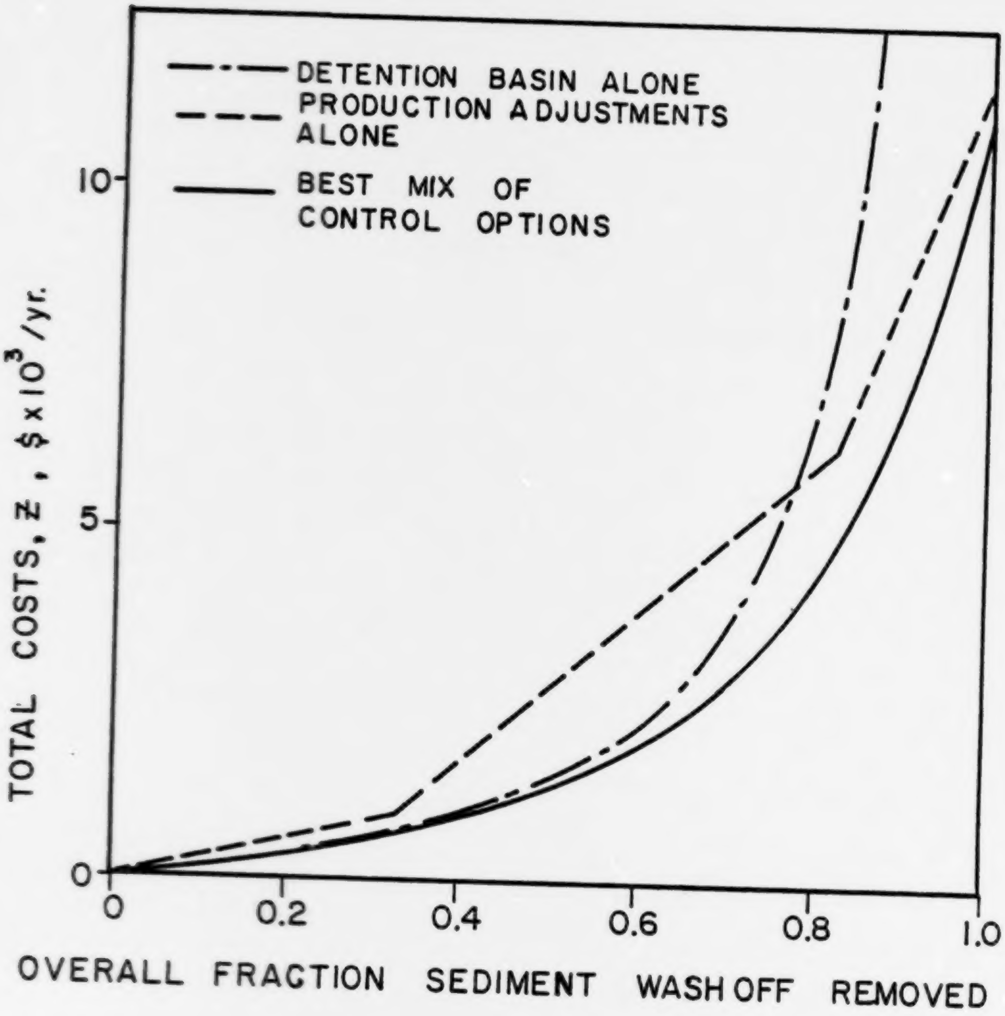

Figure 5-15. Comparison of Total Cost Curves for Most Efficient Mix of Control Options, and Each Control Option Acting Alone.

CONCLUSIONS

Relatively straightforward procedures for doing cost-effectiveness analysis have been presented. These methods can be used for selecting the optimal single treatment plant or group of treatment plants. More complex computer based optimization algorithms are not needed to solve most of these problems. Rather, emphasis is placed on devoting more effort to obtaining improved estimates of cost and performance.

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CHAPTER 6

CONCEPTS OF FINANCIAL ANALYSIS

INTRODUCTION

The previous chapter indicated that significant possibilities exist to improve economic efficiency by taking advantage of 1) economies of scale in control and distribution facilities; 2) fuller utilization of the assimilative capacity of the receiving environment; and/or 3) "excess" capacity in existing facilities. Furthermore, opportunities for mutually advantageous multi-purpose utilization of facilities could be exploited by permitting voluntary cooperation among purposes, e.g., street sweeping, which heretofore were operated as single purpose entities. The result of this economic analysis will typically indicate that the best overall solution is to use a blend of many controls, e.g., the solution set shown below:

Hypothetical Optimal Strategy for Anytown, U.S.A.

1) A tertiary sewage treatment plant for Sewer District 1.
2) Three secondary sewage treatment plants for Sewer Districts 2,3, & 4.
3) Weekly street sweeping in the commercial districts.
4) Monthly street sweeping in the other areas.
5) Sewer flushing in 20% of the combined sewer system wherein the pipes have low slopes.
6) Eighteen storm water detention ponds located in various parts of the city.
7) Co-incineration of the sewage sludge in the solid waste incinerator.
8) Swirl concentrators at eleven combined sewer overflow points.

While the above mix of controls does minimize the total cost of the pollution control program, the result may not be implementable. From a financial and economic point of view, several problems remain.

1) Overall, a cost allocation (among purposes) and sharing (among groups) arrangement must be worked out among the various purposes, centers, groups, cities, etc. The principles apply to either case. Thus, allocation will be used throughout the remainder of this report to denote the assignment of costs. The assignees will be referred to as groups.

2) Existing methods of apportioning costs among affected purposes and/or groups do not always work, i.e., they lead to unfair assignment of costs. Similar problems plague practitioners in the water resources field and in accounting, in general. The cost assignment procedure may leave some groups

paying higher costs than if they had acted alone or with some sub-group. For example, the optimal solution recommends a more expensive tertiary sewage treatment plant in Sewer District 1 whereas, without the cooperative program, they would only be required to install a less expensive secondary sewage treatment plant. More intensive treatment in District 1 was selected because of economies of scale. As a result, Districts 2 and 4 are able to install secondary rather than tertiary treatment.

3) Unresolved problems exist in deciding whether to use marginal or average costs to determine unit costs. The marginal cost approach from economics, while easy to present conceptually, is difficult if not impractical to apply for at least two reasons: (a) marginal cost data are hard to obtain, and (b) application of marginal costing will not apportion total costs. The unit costs used in the economic optimization presented in Chapter 5 assumed a specified cost allocation/sharing arrangement, e.g., the total costs of street sweeping were assumed to be 100% chargeable to pollution control. However, if a mutually acceptable cost allocation arrangement between the street department and the pollution control department could be worked out then the assumed unit cost of street sweeping would decrease significantly. This would affect the economic optimization. The reason for the indeterminacy is that the actual long-run marginal costs are unknown. Using the street sweeping example, the optimal solution states that the streets in commercial areas should be swept weekly. At present, the street department sweeps these streets every two weeks. Thus, the short-run incremental costs of adding pollution control as a purpose is simply the additional operation and maintenance costs associated with more frequent sweeping. This could be accomplished without additional capital investment by adding another shift to the street sweeping activity. The theoretical rule states that only true incremental costs are assignable. Other costs are sunk and are irrelevant. This leads to the so-called free-rider problem. In the long run, the equipment needs to be replaced. How are these costs to be divided between the two purposes? The street department might reasonably argue that pollution control, which had paid only short-run incremental costs for the past few years, should pay a large portion of the cost of the new equipment. On the other hand, pollution control might argue that they can only afford to pay these incremental costs. If charges to them were increased, they would cease street sweeping and use another control option.

Is there a "fair" way to resolve this problem without destroying the economic optimum?

4) Current Federal policies permit local agencies to cost share only the capital costs of the project. All operation and maintenance costs must be paid locally. This policy induces the local agencies to favor capital-intensive projects.

5) The cost coefficients in the economic analysis may depend on the cost allocation rule. In these cases it is necessary to solve these two problems simultaneously.

6) No government unit exists at present which is empowered to assess taxes and/or charges on the affected groups.

The succeeding sections of this chapter present procedures for systematically evaluating the financial aspects of the problem.

CURRENT PRACTICE

Environmental Engineering

The latest guidelines for financing wastewater systems are the result of a joint effort of the American Public Works Association, the American Society of Civil Engineers, and the Water Pollution Control Federation (1). Portions of these guidelines are being used by EPA (2). Their fundamental principle governing financing and charges is that:

> "The needed total annual revenue requirements of...sewage works shall be contributed by users and non-users (or by users and properties) for whose use, need and benefit the facilities of the works are provided, approximately in proportion to the cost of providing the use and the benefits of the works."

Three methods for apportioning costs are described:

1) Proportionate Share Basis-Total costs of facilities serving one party are assessed to that party alone whereas the costs of joint facilities are shared by the parties in proportion to the benefit each derives from the joint facilities. A single, e.g., flow, or multiple, e.g., flow and BOD_5 load, measure of use is used.

2) Assessed Valuation Basis-Joint costs are assigned in proportion to assessed valuation of property. This method is considered unfair because assessed valuation is not a reliable surrogate measure of use and/or benefits.

3) Incremental Cost Basis-A larger entity should increase the size and capacity of a proposed facility to permit acceptance of wastewater from a smaller entity and charge only the slight extra cost involved. The report points out that this procedure allots all of the savings from cooperation to the smaller user and may not be fair.

With regard to costing procedures, this report allocates the cost of each element of plant investment and operating expense to cost of service components. They emphasize the importance of historical practices in defining "fair" rates. Lastly, the recommended charge system is a two part method consisting of a capacity charge and a user charge.

EPA Policy

EPA Program Guidance Memorandum No. PRM 77-4 issued 3 Dec. 1976 presents guidelines for cost allocation for multiple purpose projects. Salient excerpts from PRM 77-4 are presented below:

> "As required in PRM No. 75-34 (PGM No. 61) costs of a multiple purpose project to be construction grant funded shall be allocated

among purposes. The total costs allowable for construction grant funding shall not exceed that allocated to the pollution control purpose. The pollution control allocation shall:

1) be determined by the Alternative Justifiable Expenditure (AJE) method except where unusual circumstances warrant use of other methods; and

2) in no case, exceed the cost of the least cost single-purpose pollution control alternative . . .

"The cost allocation steps are:

1) Estimate the costs of the most cost-effective single-purpose projects to obtain the same objectives as those of the multiple purpose project.

2) Determine the respective specific costs of each purpose in the multiple purpose project. The specific costs of a purpose are the sum of costs assignable to each project component exclusively serving that single purpose. An example of a specific cost would be the cost of a treatment plant included in a project designed to improve urban drainage and reduce pollution from combined sewer overflows.

3) Deduct the specific cost of each purpose in the multiple purpose project from the single-purpose project cost.

4) From the total cost of the multiple-purpose project deduct all specific costs to determine joint costs.

5) Distribute joint costs of the multiple purpose project among purposes in direct proportion to the remainders found in Step 3.

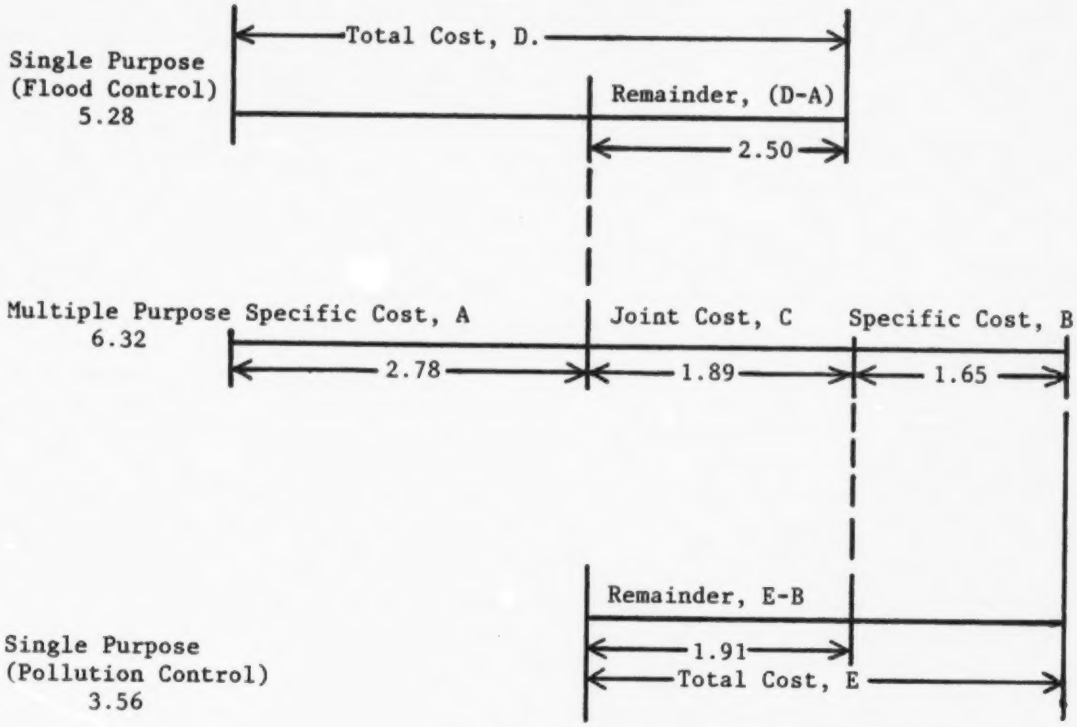
6) To obtain costs for each purpose, add the specific and the distributed joint costs for each purpose. It should be noted that none of the purposes will be assigned costs which are greater than the cost of the most cost-effective single purpose project nor less than the specific cost of the purpose."

An example application of this method is shown in Figure 6-1. Applications of EPA policy to date have been limited to two purpose cases similar to the example presented in Figure 6-1. However, as the number of purposes and/or groups involved in the cost allocation/sharing problem increases, use of the AJE and other standard methods sometimes yields solutions which are "unfair" as will be outlined later in this chapter.

TVA Studies

The original requirements for cost allocation in the Tennessee Valley Authority project in the southeastern U.S. were promulgated in 1935 (3).

ALTERNATIVE JUSTIFIABLE EXPENDITURE METHOD
COST ALLOCATION EXAMPLE
(AVERAGE ANNUAL EQUIVALENT COSTS - MILLIONS OF DOLLARS)


Total Cost, D.
Single Purpose
(Flood Control)
5.28
Remainder, (D-A)
2.50
Multiple Purpose
6.32
Specific Cost, A
Joint Cost, C
Specific Cost, B
2.78
1.89
1.65
Remainder, E-B
Single Purpose
(Pollution Control)
3.56
1.91
Total Cost, E

Flood Control Allocation $= A + \frac{D-A}{(D-A)+(E-B)} \times C$

$= 2.78 + \frac{2.50}{2.50+1.91} \times 1.89$

$= 3.85$

Pollution Control Allocation $= B + \frac{E-B}{(D-A)+(E-B)} \times C$

$= 1.65 + \frac{1.91}{2.50+1.91} \times 1.89$

$= 2.47$

Figure 6-1. Example Application of Alternative Justifiable Expenditure Method (2)

Allocation of costs was required among 1) flood control, 2) navigation, 3) fertilizer production, 4) national defense, and 5) development of power. The most controversial aspect of this problem dealt with the allocation of cost to power development. The outcome of this allocation would have a major impact on the cost of publicly produced power as compared to privately produced power which was the primary power producer at that time.

Several prominent authorities were brought in to evaluate the numerous alternatives for allocating costs. They formulated a set of five criteria which are listed below (3).

(1) The method should have a reasonable logical basis. It should not result in charging any objective with a greater investment than the fair capitalized value of the annual benefit of this objective to the consumer. It should not result in charging any objective with a greater investment than would suffice for its development at an alternate single purpose site. Finally, it should not charge any two or more objectives with a greater investment than would suffice for alternate dual purpose or multiple purpose improvement.

(2) The method should not be unduly complex. In a democracy public agencies should be subject to the understanding scrutiny of the citizen body to the maximum possible extent. The goal should always be presentation of problems and programs for public consideration in clear and simple terms.

(3) The method should be workable. It should not be dependent upon estimates which are difficult to prepare, and it should not require the use of data as to actual operation of the project which will not become available until after the passage of a considerable period of time.

(4) The method should be flexible. It should be equally applicable to an isolated project or a co-ordinated system. In the case of a system it should be applicable to both the initial and ultimate stages of development. It should be readily adaptable to changing conditions over plant or system life.

A final criterion which sometimes has been suggested but which we put forward here only for the sake of discussion is:

(5) The method should apportion to all purposes present at a multiple purpose enterprise a share in the over-all economy of the operation. This point has two implications. First, no purpose should be assessed costs as great as would suffice for alternate single purpose stream improvement. Second, no purpose should be assessed costs as great as its capitalized benefit value.

The conditions described in these five criteria can be summarized as follows:

a) The cost allocated to purpose i should not exceed its benefits, i.e.,

$$x(i) \leqq B(i) \qquad \forall\ i \varepsilon N \qquad (6\text{-}1)$$

where $x(i)$ = cost allocated to purpose i,

$B(i)$ = benefit of purpose i, and

N = set of all purposes = $\{1,2,\ldots,i,\ldots,n\}$.

b) The cost allocated to purpose i should not exceed its alternative single purpose cost, i.e.,

$$x(i) \leqq C(i) \qquad \forall\ i \varepsilon N \qquad (6\text{-}2)$$

where $C(i)$ = alternative cost if purpose i acts independently.

c) The cost allocated to any two or more purposes should not exceed their alternative cost acting as a subgroup of s of the n purposes, i.e.,

$$\sum_{i \varepsilon S} x(i) \leqq C(S) \qquad \forall\ S\ N \qquad (6\text{-}3)$$

where $C(S)$ = alternative cost if the subset s of purposes acts independently; and

S = any subset of the master set, N.

The first two criteria can be combined into a single condition,

$$x(i) \leqq \text{min. } [B(i),\ C(i)] \qquad \forall\ i \varepsilon N \qquad (6\text{-}4)$$

Notably absent from this list of criteria is the condition that the sum of the costs allocated to the various purposes equals the total cost of the project. Most of the standard methods of cost allocation require that this condition be met. However, they concluded that, based on economic theory, it is not possible to meaningfully allocate joint costs (3). The crux of the argument is that the criterion for economic efficiency requires that the marginal benefits of a purpose be compared with the marginal costs. They viewed marginal costs as being equivalent to direct costs, i.e., the costs of the elements of the project which are used solely by that purpose. Accordingly, the joint costs are viewed as sunk costs which need not be recovered.

Other Agency Studies

With the rapid growth of federal involvement in water resources projects during the 1930's and 1940's, inter-agency committees were established to standardize techniques for performing economic and cost allocation analyses. Three reports [Federal Inter-Agency Committee, 1950 (4), Inter-Agency Committee, 1958 (5); and Water Resources Council, 1962 (6)] present the following accepted guidelines:

1) The total of all benefits exceeds the total of all costs.

2) The scale of development is such as to provide maximum total benefits over total costs.

3) The benefits of adding each purpose as the last increment at least equal the separable costs of the added capacity.

With regard to cost allocation the committee recommended the separable costs, remaining benefits method which assigns to each purpose (4):

4) its separable costs; and

5) a share of the residual or remaining joint costs in proportion to the remaining benefits, i.e., the benefits (as limited by alternative costs) less the separable costs.

Before discussing these criteria, it is necessary to define "separable costs". They are defined as the difference between the cost of the multiple-purpose project and the cost of the project with the purpose omitted (4). Separable costs include direct costs and the incremental costs of changing the size of the multi-purpose cost elements. Thus, the use of separable costs represents a significant change from the TVA procedures which used direct costs (8). Separable costs are:

$$SC(i) = C(N) - C[(N) - \{i\}] \qquad \forall\ i \varepsilon N \qquad (6\text{-}5)$$

where $SC(i)$ = separable cost to group i,

$C(N)$ = total cost for the grand coalition of n groups, and

$C[(N) - \{i\}]$ = total cost for the grand coalition with group i excluded.

These additional five criteria can be represented as follows:

a) $$B(N) \geqq C(N) \qquad (6\text{-}6)$$

where $B(N)$ = total benefits for the grand coalition of n groups.

b) $$B(N) - C(N) \geqq B(W) - C(W) \qquad \forall \text{ sets, } W \qquad (6\text{-}7)$$

where $B(W)$ = total benefits for a coalition of w groups, $w \neq n$, and

$C(W)$ = total costs for a coalition of w groups, $w \neq n$; and

c) $$B(i) \geqq SC(i) \qquad \forall\ i \varepsilon N \qquad (6\text{-}8)$$

d) $$x(i) \geqq SC(i) \qquad \forall\ i \varepsilon N \qquad (6\text{-}9)$$

e) i) $$NSC = C(N) - \sum_{i \varepsilon N} SC(i) \qquad (6\text{-}10)$$

where NSC = nonseparable (joint) costs of the multipurpose project.

ii) $$\beta_i = \frac{[\min(B(i),\ C(i))] - SC(i)}{\sum_{i \varepsilon N}[\min(B(i),\ C(i)) - SC(i)]} \qquad \forall\ i \varepsilon N \qquad (6\text{-}11)$$

Thus, the total cost allocated to group i is

$$x(i) = SC(i) + \beta_i\,(NSC) \qquad \forall\, i \varepsilon N \qquad (6\text{-}12)$$

Implicit in equation (6-12) is the requirement that

$$\sum_{i \varepsilon N} x(i) = C(N) \qquad (6\text{-}13)$$

A recent article by Loughlin indicates that the above procedures are still being used (7). Indeed, the U.S. Water Resources Council is currently revising these procedures again.

Under current practice, the total cost allocated to a particular group is composed of two parts.

Let $x(i)$ = total cost allocated to $i^{\underline{th}}$ group,

$x(i)_{min}$ = minimum cost allocated to $i^{\underline{th}}$ group,

β_i = proportion of remaining costs paid by $i^{\underline{th}}$ group,

RC = Remaining costs, and

C(N) = Cost of N group project.

Then, the cost allocated to the $i^{\underline{th}}$ group is

$$x(i) = x(i)_{min} + \beta_i\,(RC) \qquad (6\text{-}14)$$

$$\text{where } RC = C(N) - \sum_{i=1}^{n} x(i)_{min}. \qquad (6\text{-}15)$$

The next section describes how $x(i)_{min}$ and β_i are determined.

a) Minimum Cost Allocated to Group i.

In water resources planning, three general procedures are used to allocate a minimum cost

a) let $x(i)_{min} = 0$, or

b) let $x(i)_{min}$ = direct or specific costs, or

c) let $x(i)_{min}$ = separable costs.

The first option is simply to set the minimum cost equal to zero and proceed to part 2. The use of facilities method which has been used in the environmental field falls into this category. In this method a single measure of use, e.g., wastewater flow, is selected. Then the total cost is allocated in proportion to the relative use of each group. The second approach deducts direct or specific costs. These costs are defined as the costs of physically identifiable facilities serving only one group. For example, in the treatment plant/pipeline example, the pipelines would be charged to the cities using them. Lastly, the separable costs are defined as the additional costs of adding a group if that group joins last.

The relevant question is to decide which of these methods is correct.

b) Choice of an Allocation Vehicle.

James and Lee list six principal vehicles that have been used for allocating joint costs (8). These vehicles and the associated value of β_i are as follows:

1) equally among groups, $\beta_i = 1/n$

2) proportional to the quantity of use the group makes of the facilities as expressed in units such as volume, flow rate, BOD level, etc., i.e.,

$$\beta_i = \frac{D_i}{\sum_{j=1}^{n} D_j} \tag{6-16}$$

where D_i = units used by group i.

3) entirely to the highest priority group within the limit of the benefit the group receives.

$$\beta_i = \min \left[1 - \sum_{j=1}^{s-1} \beta_j ; \frac{\min(B(i), C(i)) - X(i)_{\min}}{RC} \right] \tag{6-17}$$

where C(i) = cost of independent action.

s = number of groups being allocated costs

RC = remaining costs to be allocated.

4) proportional to (a) the benefit in excess of assigned separable cost derived by the given cost center; or (b) the excess cost required to provide the service by some alternative means; or (c) the smaller of the excess benefit or excess cost of the alternative project or

$$\beta_i = \frac{B_i' - x(i)_{\min}}{RC} \tag{6-18}$$

where B_i' = (a) gross benefit, or (b) alternative cost, or (c) the minimum of (a) and (b) for group i

Accounting Perspective

An important and often overriding concern is to promulgate methods in accordance with "accepted" practice. Engineering methods are composites of theory (what should work) and precedent (what has worked). Thus, it is instructive to examine the cost accounting literature (9, 10, 11). The discussion to follow draws heavily on Horngren (9).

The first point to recognize is that the cost allocation problem pervades nearly all areas of economic activity. How should the cost of a university library be apportioned among undergraduate education, graduate education, and research? How should the costs of a company's common resources such as its computer system, warehouse, legal staff, central management, or personnel office be apportioned among the products that it sells? What does it cost the U.S. Environmental Protection Agency to control sewage discharges in its western Region VIII? What part of the cost of operating the Washington EPA headquarters office is chargeable to that function? All of these questions are difficult, if not impossible, to answer precisely.

Why bother with cost allocation at all? Horngren lists several purposes of cost allocation (9):

1) Predicting economic effects of decisions. For example, what is the "cost" of street sweeping? A recent national survey by the American Public Works Association indicated that the reported unit cost of street sweeping ranges from $0.28 to $28,000 per curb mile swept (12). The primary reason for this five orders of magnitude range is differences in accounting systems and procedures for assigning common costs. For example, at the low end, the city may have included vehicular gasoline costs only. At the upper end, the city could have been depreciating their sweepers over a very short period, charging a high proportion of their garage and motor pool maintenance to street sweeping, charging a high proportion of administrative salaries to this function, and so on.

Indeed, one of the most difficult problems in evaluating water pollution control alternatives is to obtain cost data which are comparable in that a consistent set of accounting procedures was used in their determination. This problem is exacerbated by the myriad of federal programs with which cities are involved. Thus, many cities would have to update their accounting procedures to provide an adequate data base for the type of analysis presented in this report.

2) Determining income and asset valuations. This purpose is not too relevant to this discussion.

3) Ascertaining a mutually agreeable price. This purpose deals directly with the cost allocation problem being addressed in this chapter wherein charges are to be levied on participants in a multilateral arrangement. Part of the gamesmanship associated with this analysis is how to define the characteristic function for each coalition. Depending on the cost allocation rule, the groups may underestimate or exaggerate their costs. In addition, the groups could have formal or informal agreements which relate pollution

control cost allocation to costing for other purposes, e.g., water supply. Are these agreements relevant?

4) Obtaining desired motivation. The attitude of division managers towards other divisions is strongly influenced by the organization's costing procedure. For example, if the central computer facility for a city is set up as a cost center which draws a fixed portion of the revenue producing functions of the government, then other division managers, say the Director of Public Works, may view the computer division as an inescapable burden. On the other hand, if the computer center is set up as a revenue center which charges other internal divisions for services, then its director feels he is making money and will be motivated accordingly. These procedures are aimed primarily at internal motivation. However, they do provide some helpful guidance for multi-city regional organizations or for motivating departments within a city involved in a multi-purpose pollution control program.

Uniqueness of each Problem

The related problems of determining "true costs" and "fair and equitable" cost allocations are very difficult ones. While the theoretical background can assist groups in their deliberations, they will ultimately have to decide what is "fair." This section summarized cost allocation/sharing procedures which have evolved in the related water resources field over the past 40-50 years. Next, parallel developments in cooperative N-person game theory will be described. These notions from game theory provide a more generalized definition of "fairness". Lastly, recommendations regarding generalized procedures for cost allocation will be presented.

COOPERATIVE N-PERSON GAME THEORY

Parallel to, and completely independent of water resources activities, game theorists have been wrestling with related problems of fairness and equity in allocating costs or savings among members of a group who voluntarily agree to cooperate if each one feels that it is advantageous. Rapoport presents a good introduction to the basic ideas (13). Luce and Raiffa, and Owen are good sources for intermediate and advanced material on the subject (14,15).

Characteristic Functions and Super (Sub)-Additivity

An N-Person game (N, v), in characteristic function form, consists of a set, N = 1, 2, ..., n, of N players along with the characteristic function, v, which assigns the real number v(S), to each non-empty subset, S, of players. As an example, recall the three city example from the introductory chapter wherein the value of the characteristic function is defined as

v(S) = minimum cost combination of treatment plants
and pipelines for the S member coalition.

The resulting characteristic function values are shown below:

$$v(1) = \$2,300,000, \quad v(2) = \$1,600,000, \quad v(3) = \$2,300,000$$

$$v(12) = \$3,500,000, \quad v(13) = \$4,600,000, \quad v(23) = \$3,650,000$$

$$v(123) = \$5,560,000$$

As pointed out in our earlier work, alternative definitions of the characteristic function can be used (17). In defining the optimal solution for coalition S, it is necessary to unambiguously define the behavior of the complementary coalition N-S. As an example, suppose the three cities in the above example had a specified amount of assimilative capacity available to them. However, no explicit ownership rights are specified. Then, in order to calculate v(S), it is necessary to define how this assimilative capacity is to be apportioned. The following procedures could be used:

1) Somehow apportion the available assimilative capacity among the three cities, e.g., each one gets 1/3 of the total.

2) Group S can have any remaining assimilative capacity after N-S has used all that it wants.

3) Group S can use as much assimilative capacity as it wants. The resulting values of the characteristic function will differ widely depending on the selected definition.

Revenue (costs) games are super (sub) additive, i.e.,

$$v(S) + v(T) \leqq v(S\ T) \qquad \text{for } S\ T = \phi \tag{6-19}$$

for revenue games, and

$$v(S) + v(T) \geqq v(S\ T) \qquad \text{for } S\ T = \phi \tag{6-20}$$

for cost games. ϕ is the empty set. S and T are any two subsets of N. If the economic optimization has been done correctly, then the super (sub) additivity condition should automatically be satisfied. For the three city cost example,

$$v(S) + v(T) \geqq v(S\ T) \qquad \text{for } S\ T = \phi,$$

i) $v(1) + v(2) \geqq v(12)$
$2,300,000 + 1,600,000 > 3,500,000$ OK

ii) $v(1) + v(3) \geqq v(13)$
$2,300,000 + 2,300,000 = 4,600,000$ OK

iii) $v(2) + v(3) \geqq v(23)$
$1,600,000 + 2,300,000 > 3,650,000$ OK

iv) $v(1) + v(23) \geqq v(123)$
$2,300,000 + 3,650,000 > 5,560,000$ OK

v) $v(2) + v(13) \geq v(123)$
$1{,}600{,}000 + 4{,}600{,}000 > 5{,}560{,}000$ OK

vi) $v(3) + v(12) \geq v(123)$
$2{,}300{,}000 + 3{,}500{,}000 > 5{,}560{,}000$ OK

Satisfaction of super (sub) additivity is a prerequisite for voluntary cooperation. If it is not satisfied, then at least one coalition exists for which savings (costs) would be higher (lower) if the members did not form the coalition. But this is impossible if the optimal solution, either maximum savings or minimum costs, has been found. At worst, no savings or lower costs would result when the coalition formed in which case the coalition is said to be inessential, e.g.,

$$v(1) + v(3) = v(13).$$

Cities 1 and 3 are too far apart to justify piping wastes from City 1 to a regional plant at City 3.

Fairness Criteria

Game theorists state three general axioms which a "fair" solution should satisfy. In the context of this report, fairness is defined as a set of solutions satisfying certain behavioral axioms. No broader notion of fairness is intended. They are presented in Table 6-1.

Criterion	GAME	
	Maximize Savings	Minimize Costs
1) Individual Rationality for all i	$x(i) \geq v(i)$	$x(i) \leq v(i)$
2) Group Rationality	$\sum_{i \in N} x(i) = v(N)$	$\sum_{i \in N} x(i) = v(N)$
3) Sub-Group Rationality for all $S \subset N$	$\sum_{i \in S} x(i) \geq v(S)$	$\sum_{i \in S} x(i) \leq v(S)$

Table 6-1. Equity Criteria for Cooperative Games.

The first criterion states that the savings (costs) assigned to the i^{th} group, x_i, must be at least (no more than) the savings (costs) they would receive acting independently. The second criterion states that the total savings (costs) must be allocated among the N groups. All solutions satisfying criteria 1 and 2 are called imputations. The third criterion extends the first criterion by insisting that the payoff (cost) to each group be at least (no more than) the savings (costs) they would receive in any subcoalition, S, contained in N. Solutions satisfying all three conditions constitute the core of the game. For super (sub) additive games, the set of imputations is nonempty but the core may be empty.

Graphical representations can be used for two and three person games. As an example of a two-person game, consider the EPA example (Figure 6-1) of a dual purpose flood (F) control and pollution (P) control cost game. The values (in $\$ \times 10^6$) of the characteristic function are

$$v(F) = 5.28; \; v(P) = 3.56$$

$$v(FP) = 6.32$$

This game is sub-additive, i.e.,

$$v(F) + v(P) > v(FP).$$

Denoting the costs assigned to flood control and pollution control as x(F) and x(P), the solution space is plotted in Figure 6-2. The conditions are

$$\begin{array}{lcl} x(F) & & \leqq 5.28 \\ & x(P) & \leqq 3.56 \\ x(F) & + \; x(P) & = 6.32 \end{array}$$

For two person games the set of imputations and the core are identical since there are no additional sub-groups. In this case, the set of admissible solutions lies along the line $\overline{AB}$. The question of picking the fairest solution is deferred until later.

Consider the three city cost game as an example of a three dimensional case. The conditions are as follows (all costs in $\$ \times 10^6$)

$$v(1) = 2.30, \; v(2) = 1.60, \; v(3) = 2.30$$

$$v(12) = 3.50, \; v(13) = 4.60, \; v(23) = 3.65$$

$$v(123) = 5.56$$

$$\begin{array}{llll} x(1) & & & \leqq 2.30 \\ & x(2) & & \leqq 1.60 \\ & & x(3) & \leqq 2.30 \\ x(1) & + \; x(2) & & \leqq 3.50 \\ x(1) & & + \; x(3) & \leqq 4.60 \\ & x(2) & + \; x(3) & \leqq 3.65 \\ x(1) & + \; x(2) & + \; x(3) & = 5.56 \end{array}$$

The first three conditions stipulate upper bounds on x(i).
The next three conditions stipulate lower bounds on x(i).

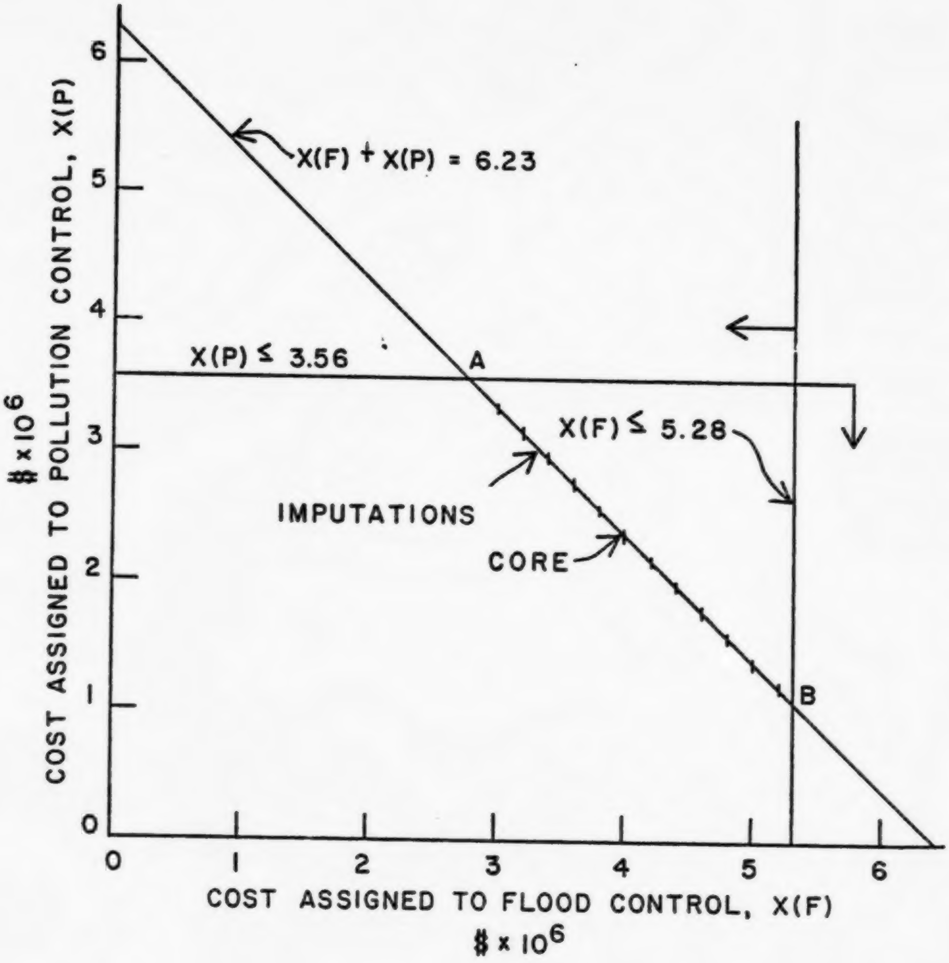


Figure 6-2. Imputations and Core for Two Person Game.

$x(1) + x(2) \leqq 3.50$ implies that

$$x(3)_{min} = v(123) - v(12) = 2.06$$

Likewise $x(2)_{min} = v(123) - v(13) = 0.96$, and

$$x(1)_{min} = v(123) - v(23) = 1.91.$$

Thus, the admissible solutions must satisfy

$$1.91 \leqq x(1) \leqq 2.30$$

$$0.96 \leqq x(2) \leqq 1.60$$

$$2.06 \leqq x(3) \leqq 2.30$$

A plot of these conditions is shown in Figure 6-3. The set of imputations is the triangle as shown. Note that the large triangle DEF is equilateral. It has the nice property that the sum of the coordinates of any selected point on the triangle is 5.56. Because of this special property, it is possible to use a special kind of graph paper called isometric paper to plot this three dimensional plot in two dimensions as shown in Figure 6-4.

With respect to the core, three cases can occur:

1) core is convex
2) core exists but is non-convex
3) core is empty

A game has a convex core if

$$v(S) + v(T) \geqq v(S\ T) + v(S\ T) \quad \text{for } S\ T \neq \phi \tag{6-21}$$

for a cost game, and

$$v(S) + v(T) \leqq v(S\ T) + v(S\ T) \quad \text{for } S\ T \neq \phi \tag{6-22}$$

for a savings game.

As a generality, the more attractive (higher savings or lower costs) that the game is, the greater the chance that the core is convex. Conversely, the less attractive the game is, the greater the chance that the core is empty. Only the additional possibilities associated with $S\ T \neq \phi$ need to be checked. The cases where $S\ T = \phi$ were checked as part of the super (sub) additivity analysis. For the three city example, the additional conditions to be checked are:

$$\begin{array}{llll} C(12) + C(13) & \geqq & C(123) + C(1) & \\ 3.50 + 4.60 & \geqq & 5.56 + 2.30 & \text{OK} \end{array}$$

$$\begin{array}{llll} C(12) + C(23) & \geqq & C(123) + C(2) & \\ 3.50 + 3.65 & \geqq & 5.56 + 1.60 & \text{NO} \end{array}$$

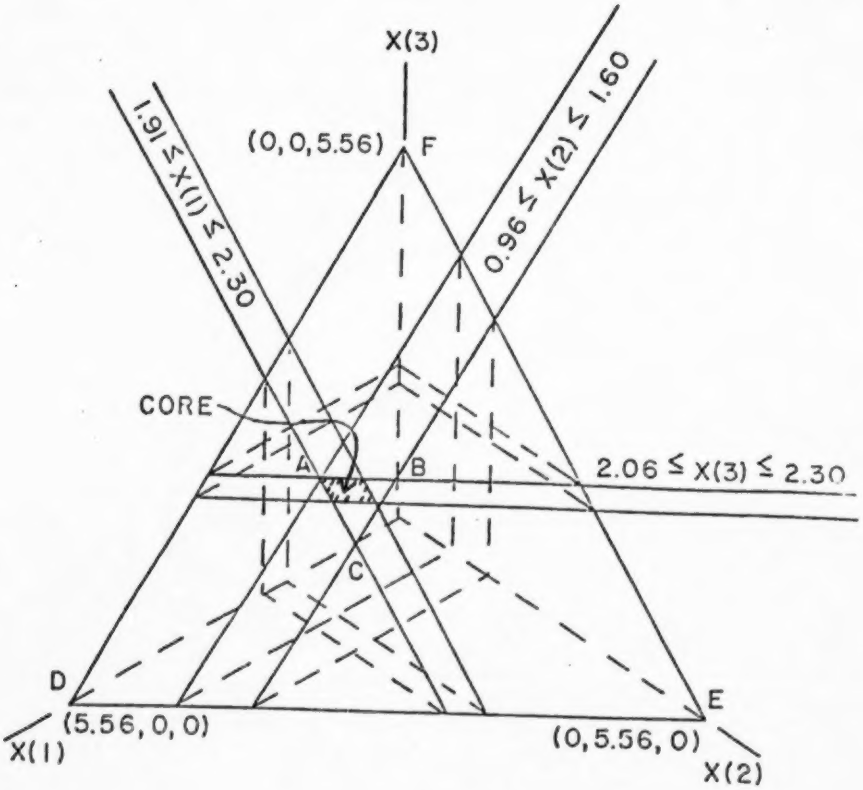

X(3)
(0,0,5.56)
F
1.91 ≤ X(1) ≤ 2.30
0.96 ≤ X(2) ≤ 1.60
CORE
A
B
C
2.06 ≤ X(3) ≤ 2.30
D
E
(5.56,0,0)
X(1)
(0,5.56,0)
X(2)

Figure 6-3. Three Dimensional Plot of Three Person Cost Game.

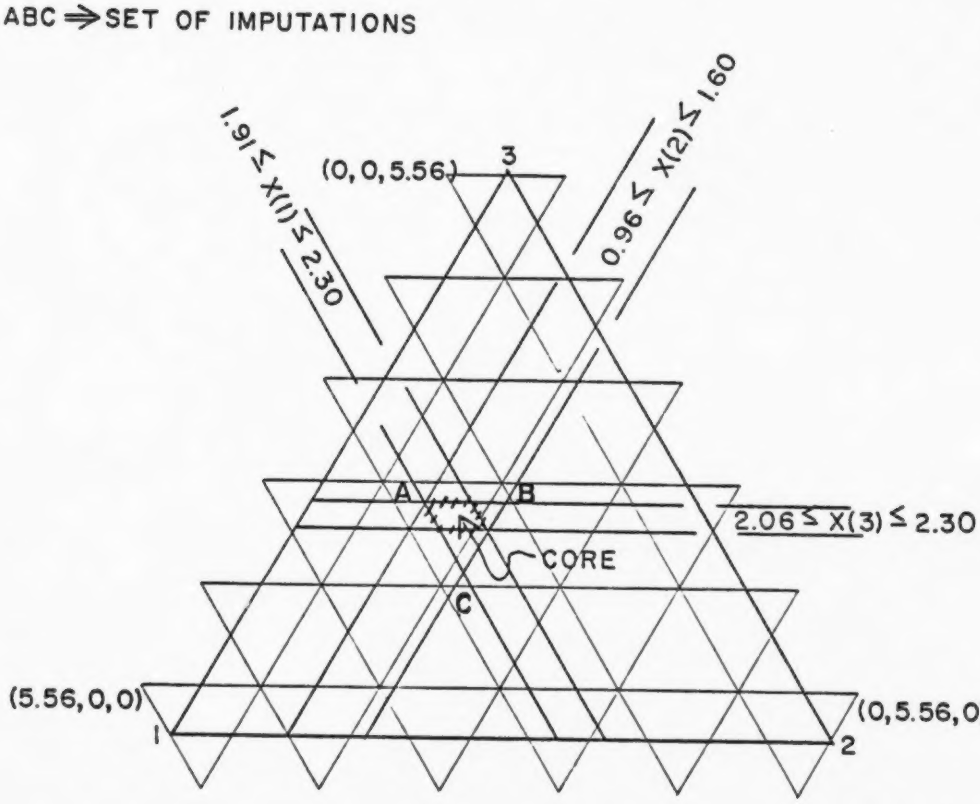

ABC ⇒ SET OF IMPUTATIONS
1.91 ≤ X(1) ≤ 2.30
0.96 ≤ X(2) ≤ 1.60
(0, 0, 5.56)
3
A
B
2.06 ≤ X(3) ≤ 2.30
CORE
C
(5.56, 0, 0)
1
(0, 5.56, 0)
2

Figure 6-4. Solution Using Isometric Paper.

$$C(13) + C(23) \geq C(123) + C(3)$$
$$4.60 + 3.65 \geq 5.56 + 2.30 \qquad \text{OK}$$

Thus, this core is non-convex (barely). As will be shown later, convex games have especially nice solution properties.

Unique Solution Notions

The "fairness" conditions significantly delimit the set of admissible solutions. However, additional criteria are needed to select a unique solution. Numerous unique solution notions exist in the game theory literature. However, the two most popular solution notions are the Shapley value (16) and the nucleolus (17).

...Shapley Value

Shapley (16) argues that each group should get (pay) the incremental value (cost) of adding them to the coalition, i.e., the incremental charge due to the $i^{\underline{th}}$ group is

$$v(S) - v(S - \{i\}).$$

However, we don't know in which order coalitions will form. In the three city example, six coalition formation sequences exist:

123	213	312
132	231	321

Examination of the incremental costs for City 1 reveals that if he joins

1st, he pays $v(1) - v(0) = \$2,300,000$,

2nd, after 2, he pays $v(12) - v(2) = \$1,900,000$,

2nd, after 3, he pays $v(13) - v(3) = \$2,300,000$,

3rd, after 23, he pays $v(123) - v(23) = \$1,910,000$.

Similarly, for City 2, if he joins

1st, he pays $v(2) - v(0) = \$1,600,000$,

2nd, after 1, he pays $v(12) - v(1) = \$1,200,000$,

2nd, after 3, he pays $v(23) - v(3) = \$1,350,000$, or

3rd, after 13, he pays $v(123) - v(13) = \$960,000$.

Lastly, for City 3, if he joins

1st, he pays $v(3) - v(0) = \$2,300,000$, 2nd, after 1, he pays $v(13) - v(1) = \$2,300,000$,

2nd, after 2, he pays $v(23) - v(2) = \$2,050,000$, or

3rd, after 12, he pays $v(123) - v(12) = \$2,060,000$.

Thus, we have the following preferences:

City	Preferred Ordering
1	213 or 231
2	132
3	231 or 123

In a given case study, each city naturally prefers that coalition formation sequence which is least costly. Typically each one would like to join at or near the end. However, someone must go first so an impasse is reached. To resolve this problem, Shapley offered the following compromise. Assume all coalition formation sequences are equi-likely. Then, the probability that city i will be the last one to join an S member coalition is $\alpha_i(S)$ where

$$\alpha_i(S) = \frac{(s-1)!(n-s)!}{n!} \qquad (6\text{-}23)$$

For example, for $i = 1$, $n = 3$

$$\alpha_1(1) = \frac{0!2!}{3!} = 1/3$$

$$\alpha_1(12) = \frac{1!1!}{3!} = 1/6$$

$$\alpha_1(13) = \frac{1!1!}{3!} = 1/6$$

$$\alpha_1(123) = \frac{2!0!}{3!} = \underline{1/3}$$

$$1.0$$

Each city gets (pays) the incremental value (cost) he brings to coalitions expected out over all coalition formation sequences, i.e.,

$$\phi_i = {}_S\sum{}_N \; \alpha_i(S)\,[v(S) - v(S - \{i\})] \qquad (6\text{-}24)$$

For the three city cost game, the Shapley values are

$$\phi_1 = 1/3\,[v(1)] + 1/6[v(12) - v(2)] + 1/6[v(13) - v(3)] + 1/3[v(123) - v(23)]$$

$$= 1/3\,(2,300,000) + 1/6[1,900,000] + 1/6\,[2,300,000] + 1/3\,[1,910,000]$$

$$\phi_1 = 2,100,000$$

Likewise, $\phi_2 = 1{,}280{,}000$, and $\phi_3 = 2{,}180{,}000$

and $\sum_{i=1}^{3} \phi_i = \$5{,}560{,}000$ so that total costs are covered.

If the core of the game is convex, then the Shapley value is in the center of the core (18). However, for nonconvex games, the Shapley value may fall outside the core - a very undesirable property.

...Nucleolus

The nucleolus (17) maximizes the minimum happiness of any group. The measure of the happiness of coalition S, called e_s, is the savings realized by the members of S from cooperation.

Let e_1 denote the minimum savings of any S.

Then

$$e_1 = \min_{S\ N}[C(S) - \sum_{i \in S} x_i] \tag{6-25}$$

The nucleolus is found by solving the following linear programming problem.

maximize e_1

subject to

$$e_1 + x_i \leqq C(i) \qquad \forall i = 1,2,\ldots,n$$

$$e_1 + \sum_{i \in S} x_i \leqq C(S) \qquad \forall S\ \ N$$

$$\sum_{i \in N} x_i = C(N)$$

The nucleolus is calculated by sequentially solving for e_1, then e_2, e_3, etc. where e_i is the $i^{\underline{th}}$ smallest savings to any coalition. The nucleolus is found after at most n - 1 linear programs. The values of the x_i are established sequentially as the successive linear programs are solved. It satisfies all of the core conditions, always exists and is unique. For the EPA dual purpose example

$$C(F) = 5.28 \qquad C(P) = 3.56$$

$$C(FP) = 6.32$$

The nucleolus is found by solving the following linear program

maximize e_1

subject to

$$e_1 + x_F \leqq 5.28$$

$$e_1 + x_p \leqq 3.56$$

$$x_F + x_P = 6.32 \tag{6-26}$$

$$x_F,\ x_P \geqq 0$$

Figure 6-5 shows a plot of the constraints expressed as a function of x_F and x_P only. We will solve for e_1 by trial and error.

Trial 1. Pick the solution favoring P, i.e., (5.28, 1.04) Substituting into conditions 6-26 yields $e_1 = 0$.

Trial 2. Pick the solution favoring F, i.e., (2.76, 3.56) Substituting into conditions 6-26 yields $e_1 = 0$ again.

Trial 3. (5.00, 1.32), $e_1 = .28$

Trial 4. (4.50, 1.82), $e_1 = .78$

Trial 5. (4.00, 2.32), $e_1 = 1.24$

Trial 6. (3.50, 2.82), $e_1 = .74$

Trial 7. (3.00, 3.32), $e_1 = .24$

From the trial and error procedure, it is apparent that the optimal solution is to equalize the excess. Thus, all three constraints must hold as equalities and a unique solution is obtained to

$$\begin{aligned} e_1 + x_F \phantom{{}+x_P} &= 5.28 \qquad (a) \\ e_1 \phantom{{}+x_F} + x_P &= 3.56 \qquad (b) \\ x_F + x_P &= 6.32 \qquad (c) \end{aligned} \tag{6-27}$$

A graphical solution can be obtained by eliminating e_1 from equations (a) and (b) to yield

$$x_F - x_P = 1.72 \tag{6-28}$$

Plotting this equation onto Figure 6-5 yields the optimal solution of (4.02, 2.30) with $(e_1)_{max} = 1.26$. This solution is in the center of the core. Thus, for two person games, the nucleolus can be found graphically by plotting the core constraints and finding the intersection of that line with

$$x_1 - x_2 = c_1 - c_2, \tag{6-29}$$

or this solution may be found algebraically by solving equation set (6-27). Next, let us find the nucleolus for the three city cost game.

$$\text{maximize } e_1$$

$$\begin{aligned} \text{subject to } e_1 + x_1 \phantom{{}+x_2} &\leqq 2.30 \\ e_1 \phantom{{}+x_1} + x_2 &\leqq 1.60 \end{aligned}$$

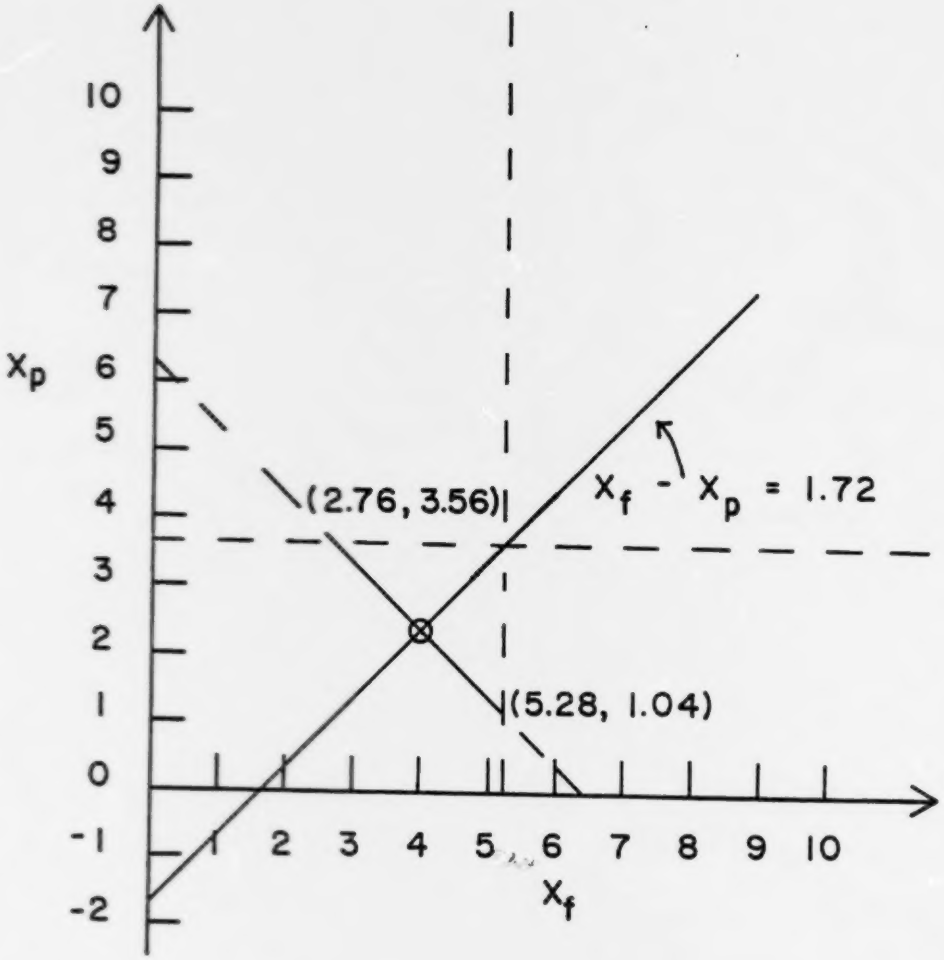


Figure 6-5. Core for the Flood Control, Pollution Control Game.

$$
\begin{array}{llllll}
e_1 & & & + x_3 & \leqq & 2.30 \\
e_1 & + x_1 & + x_2 & & \leqq & 3.50 \\
e_1 & + x_1 & & + x_3 & \leqq & 4.60 \\
e_1 & & + x_2 & + x_3 & \leqq & 3.56 \\
 & x_1 & + x_2 & + x_3 & = & 5.56 \\
 & x_1, & x_2, & x_3 & \geqq & 0
\end{array}
\qquad (6\text{-}30)
$$

We want to maximize the minimum surplus available to coalitions C(1), C(2), C(3), C(12), C(13), and C(23). The surplus of C(123) does not concern us since it is always zero. The surplus of a coalition is the difference between its characteristic function cost and the allocated cost of member (S) of the coalition. For example, the surplus of 23 is

$$e_{23} = C(23) - x_2 - x_3 \qquad (6\text{-}31)$$

where e_{23} = surplus of coalition 23,

x_2, x_3 = assigned cost of player 2 or player 3, and

C(23) = characteristic function cost.

When calculating the nucleolus by hand, an initial cost allocation is assumed. The surplus' are next calculated for the assumed cost allocation. We will assume that $x_1 = 2.0$, $x_2 = 1.56$, $x_3 = 2.0$ to start. The surplus of each coalition is

$$e_1 = C(1) - x(1) = 2.3 - 2.0 = .3,$$

$$e_2 = C(2) - x(2) = 1.6 - 1.56 = .04,$$

$$e_3 = C(3) - x(3) = 2.3 - 2.0 = .3,$$

$$e_{12} = C(12) - x(1) - x(2) = 3.5 - 3.56 = -.06,$$

$$e_{13} = C(13) - x(1) - x(3) = 4.6 - 4.0 = .6, \text{ and}$$

$$e_{23} = C(23) - x(2) - x(3) = 3.56 - 3.56 = 0.$$

The minimum surplus is e_{12} with a surplus of [-.06]. We want to maximize this surplus. The surplus of coalition 12 is increased by taking some of coalition 3's surplus. The difference between e_3 and e_{12} is .36. The most we can decrease e_3 by is .18. Then e_3 and e_{12} will have the same surplus. The new surplus' are calculated next for $x_3 = 2 + .18 = 2.18$, $x_1 = 2 - .09 = 1.91$ and $x_2 = 1.56 - .09 = 1.47$. Players 1 and 2 share equally in the increase of coalition 12's surplus.

The surplus of each coalition is now

$$e_1 = 2.3 - 1.91 = .39,$$

$$e_2 = 1.6 - 1.47 = .13,$$

$$e_3 = 2.3 - 2.18 = .12,$$

$$e_{12} = 3.5 - 3.38 = .12,$$

$$e_{13} = 4.6 - 4.09 = .51, \text{ and}$$

$$e_{23} = 3.56 - 3.65 = -.09.$$

The minimum surplus is e_{23}. We maximize e_{23} by taking part of e_1. The most we can increase e_{23} by is .24. At this new surplus e_1 equals e_{23}. Player 2 receives all of the increase in coalition 23's surplus. Player 3 cannot receive any of coalition 23's additional surplus since we already maximized player 3's surplus on the previous iteration.

The new assigned costs are $x_1 = 1.91 + .24 = 2.15$, $x_2 = 1.47 - .24 = 1.23$, $x_3 = 2.18$.

The surplus of each coalition is now

$$e_1 = 2.3 - 2.15 = .15,$$

$$e_2 = 1.6 - 1.23 = .37,$$

$$e_3 = 2.3 - 2.18 = .12,$$

$$e_{12} = 3.5 - 3.38 = .12,$$

$$e_{13} = 4.6 - 4.33 = .27, \text{ and}$$

$$e_{23} = 3.56 - 3.41 = .15.$$

The nucleolus is now calculated. x_1 and x_3 are now fixed, thereby fixing x_2, since $x_2 = C(123) - x_1 - x_3$. The final cost allocation is $X_N = [2.15, 1.47, 2.18]$, the same as the Shapley value.

CONVENTIONAL AND GAME THEORETIC CRITERIA (19)

Summary of Conventional Criteria

Workable procedures for examining the efficiency/equity aspects of environmental problems in the U.S. are needed to support the current multi-billion dollar per year control program. Based on extensive experience in the water resources and related fields, an acceptable procedure should satisfy the following conditions. It remains to be considered whether any procedure satisfies all of these conditions and is still simple, workable, and flexible.

...Economic Efficiency

1. Total benefits equal or exceed total costs, i.e.,

$$B(N) \geqq C(N) \tag{6-32}$$

2. The scale of development is such as to provide maximum total benefits minus total costs, i.e.,

$$B(N) - C(N) > B(W) - C(W) \qquad \forall \text{ sets, } W \tag{6-33}$$

3. The marginal benefits of adding each group as the last increment at least equal the separable costs of the added capacity, i.e.,

$$MB(i) \geqq SC(i) \qquad \forall\, i \varepsilon N \tag{6-34}$$

...Equity

1. The cost allocated to group i should not exceed its benefits nor its alternative cost, i.e.,

$$x(i) \leqq \min[B(i),\, C(i)] \qquad \forall\, i \varepsilon N \tag{6-35}$$

2. The costs allocated to any two or more groups should not exceed their alternative cost acting as a subgroup of s out of a total of n groups, i.e.,

$$\sum_{i \varepsilon S} x(i) \leqq C(S) \qquad \forall\, S \varepsilon N \tag{6-36}$$

3. Each group i pays at least its separable costs, i.e.,

$$x(i) \geqq SC(i) \qquad \forall\, i \varepsilon N \tag{6-37}$$

4. Each group pays a share of the residual joint (nonseparable) costs in proportion to the remaining benefits (as limited by alternative costs) less the separable costs. The nonseparable costs (NSC) and apportioning factor (β_i) need to be defined.

$$NSC = C(N) - \sum_{i \varepsilon N} SC(i), \tag{6-38}$$

$$\beta(i) = \frac{[\min\,(B(i),\, C(i))] - SC(i)}{\sum_{i \varepsilon N} [\min\,(B(i),\, C(i)) - SC(i)]} \qquad \forall\, i \varepsilon N \tag{6-39}$$

where $$\sum_{i \varepsilon N} \beta(i) = 1.0 \tag{6-40}$$

This condition can be stated as

$$x(i) = SC(i) + \beta(i)\,(NSC) \qquad \forall\, i \varepsilon N \tag{6-41}$$

5. The sum of the costs allocated to all groups equals the total cost, i.e.,

$$\sum_{i\varepsilon N} x(i) = C(N). \tag{6-42}$$

From equation 6-38,

$$C(N) = NSC + \sum_{i\varepsilon N} SC(i). \tag{6-43}$$

Substituting eqn. 6-41 into eqn. 6-38 yields

$$C(N) = NSC + \sum_{i\varepsilon N} [x(i) - \beta(i)\ (NSC)]. \tag{6-44}$$

From eqn. 6-40, if $\sum_{i\varepsilon N} \beta(i) = 1$, then eqn. 6-42 is automatically satisfied.

Relation to Game Theory

Overall, seven conditions need to be satisfied. The reader familiar with cooperative n-person game theory will recognize that conditions 6-35, 6-36, and 6-42 constitute the core of a cost game. However, there is one defect in condition 6-36. Condition 6-35 recognizes the possibility that, for a single purpose, B(i) could be less than C(i). Thus, it is proper to use the minimum of B(i) and C(i) as the maximum justifiable alternative investment. The same argument should apply to condition 6-36 since it is possible for $B(S) < C(S)$ for any $S\varepsilon N$. However, because of condition 6-32, $B(N) \geqq C(N)$. Thus, condition 6-36 needs to be modified to

$$\sum_{i\varepsilon S} x(i) \leq \min[B(S), C(S)] \qquad \forall\ S\varepsilon N \tag{6-45}$$

to render conditions 6-35, 6-42 and 6-45 equivalent to conditions for the core of a cost game. Condition 6-33 is automatically satisfied if the game is subadditive. It does serve as a reminder that explicit proof is needed that the n purpose coalition is optimal. The similarity of conventional criteria and the core is striking. Of course, the vital link is criterion 6-36 which was stated by the TVA experts in 1942 but which has never been used subsequently in the water resources field until the recent introduction of core theory.

Finding the Minimum Cost Allocation to Purpose i

The next question to be resolved is whether to deduct direct or separable costs prior to allocating the remaining costs. Accepting criteria 6-37 implies that separable costs must be deducted first. This condition is disturbing to many practitioners who routinely use direct costing techniques. The defect in direct costing is that it fails to incorporate the external benefits (or costs) caused by the project elements. As a very simple example consider two cities of equal size located along a river. Two wastewater control options are being considered: two treatment plants, or a single large plant at downstream city 2 with a pipe from city 1 to city 2. The costs are as follows:

Alternative 1

Treatment plant at city 1 = \$10

Treatment plant at city 2 = \$10

$$\overline{\$20}$$

Alternative 2

Pipe from city 1 to city 2 = \$ 4

Treatment plant at city 2 = \$14

$$\overline{\$18}$$

Using the direct cost method, city 1 pays for the pipe and a prorated share of the treatment plant, or

$$x(1) = 4 + 1/2(14) = 11 > C(1).$$

He objects and drops out. Thus, using direct costs may violate conditions (6-35) and/or (6-45).

The use of separable costs evolves from the condition (6-34) for economic efficiency wherein additional purposes would be added as long as their incremental benefits are at least equal to the incremental costs of serving them. Accepted procedures define the separable cost to purpose i as the increment in cost given that this purpose joins last. The apparent reasoning is that this is the most favored order in which to join and thereby represents the minimum cost that this purpose should pay. However, in game theory, it is known that

$$C(N) - C(N - \{i\}) \leqq C(S) - C[(S) - \{i\}] \qquad \text{for } \forall\ S \quad N \qquad (6\text{-}46)$$

only if the game is convex (Shapley, 1971). Figure 6-6 depicts the standard presentation of optimality conditions. The game is convex in the range $0 \leqq Q \leqq Q_1$, i.e., marginal costs are nonincreasing. However, the optimal solution dictates expansion to Q^*. Thus, in this case, and quite often, the lowest incremental costs is not $C(N) - C[(N) - \{i\}]$. Consequently, the economic efficiency criteria that $MB(i) \geqq SC(i)$ is too restrictive if $SC(i)$ is defined as $C(N) - C(N - \{i\})$. Then, what is the minimum cost assignable to group i?

Consider the following simple example of a three city cost game.

$$B(1) = 6,\ B(2) = 7,\ B(3) = 6$$

$$C(1) = 2,\ C(2) = 4,\ C(3) = 6$$

$$C(12) = 5,\ C(13) = 6,\ C(23) = 6$$

$$C(123) = 6$$

Inspection of this game indicates that $B(i) \geqq C(i)$ for all i and $B(S) \geqq C(S)$

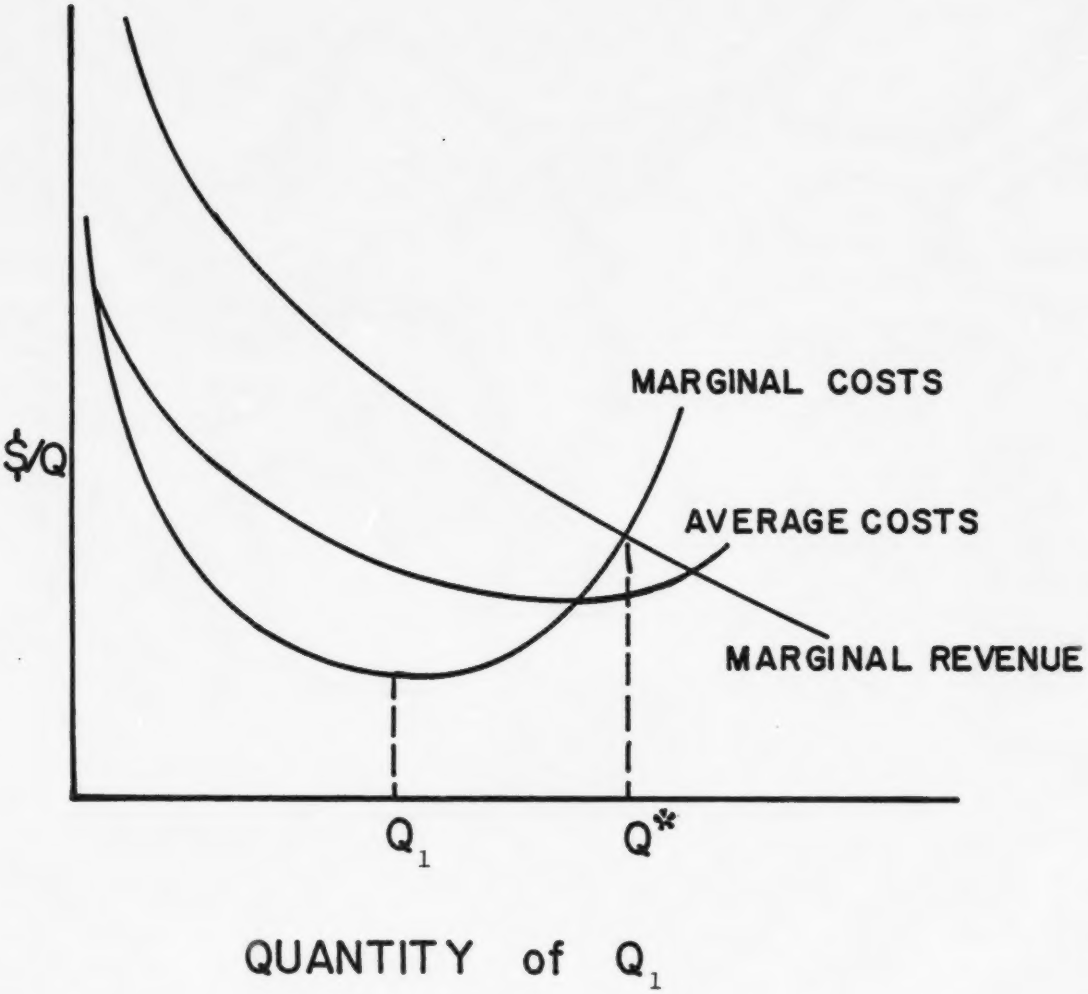


Figure 6-6. Conventional Determination of Optimal Level of Output, Q*

for all S N. Thus, there is no further need to consider benefits. For this game to be subadditive

$$C(1) + C(2) \geqq C(12), \qquad (6\text{-}47)$$

$$C(1) + C(3) \geqq C(13), \qquad (6\text{-}48)$$

$$C(2) + C(3) \geqq C(23), \qquad (6\text{-}49)$$

$$C(1) + C(23) \geqq C(123), \qquad (6\text{-}50)$$

$$C(2) + C(13) \geqq C(123), \text{ and} \qquad (6\text{-}51)$$

$$C(3) + C(12) \geqq C(123). \qquad (6\text{-}52)$$

For this example, the maximum value of C(123) = 8.0 because of condition (6-50). At that value of C(123) the coalition is inessential, i.e., C(1) + C(23) = C(123). If C(123) exceeds 8.0 it would be economically inefficient to form this coalition.

This subadditive game has a core since

$$C(12) + C(13) + C(23) \geqq 2C(123) \qquad (6\text{-}53)$$

Furthermore, since

$$C(12) + C(13) \geqq C(123) + C(1), \qquad (6\text{-}54)$$

$$C(12) + C(23) \geqq C(123) + C(2), \text{ and} \qquad (6\text{-}55)$$

$$C(13) + C(23) \geqq C(123) + C(3) \qquad (6\text{-}56)$$

the game is convex if C (123) ≦ 6.0. For this special case

$$C(N) - C(N-\{i\}) \leqq C(S) - C(S-\{i\}) \qquad \forall\ S\ \ N \qquad (6\text{-}57)$$

Accordingly, separable costs, as conventionally defined, would be the minimum incremental costs, as confirmed below in Table 6-2 for this example. However, for C(123) = 8.0, the minimum incremental cost occurs if some of the groups join second, not last. In this case, the economic criterion would incorrectly compare incremental benefits with incremental costs which were not the lowest. Figures 6-7 and 6-8 show the core of the original convex game with C(123) = 6 and a modified game with C(123) = 8.0. For games with cores, the minimum feasible (within the core) cost which can be assigned to each group is found by solving the following linear programming problem for group i (22):

$$\text{minimize } x(i)$$

$$\text{subject to } x(i) \leqq \min\,[B(i), C(i)] \qquad \forall\ i \varepsilon N,$$

$$\sum_{i \varepsilon S} x(i) \leqq \min[B(S), C(S)] \qquad \forall\ S\ \ N, \qquad (6\text{-}58)$$

$$\sum_{i\varepsilon N} x(i) \geq C(N), \text{ and}$$

$$x(i) \geq 0 \qquad\qquad \forall\ i\varepsilon N.$$

The constraint set simply delimits the core of the game so that the minimum value of x(i) is the lowest feasible cost which can be assigned to group i without violating the core conditions. For the three person game, it is the minimum costs shown on Figures 6-7 and 6-8.

Group	Incremental Cost of Joining				
	First	Second After Group		Third	With C(123) =
i		j	k	6.0^a	8.0^b
1	2	1	0	0	2.0
2	4	3	0	0	2.0
3	6	4	2	1	3.0

[a] convex game

[b] nonconvex game with a core

Table 6-2. Incremental Costs for Three Group Game with Variable Costs for Grand Coalition.

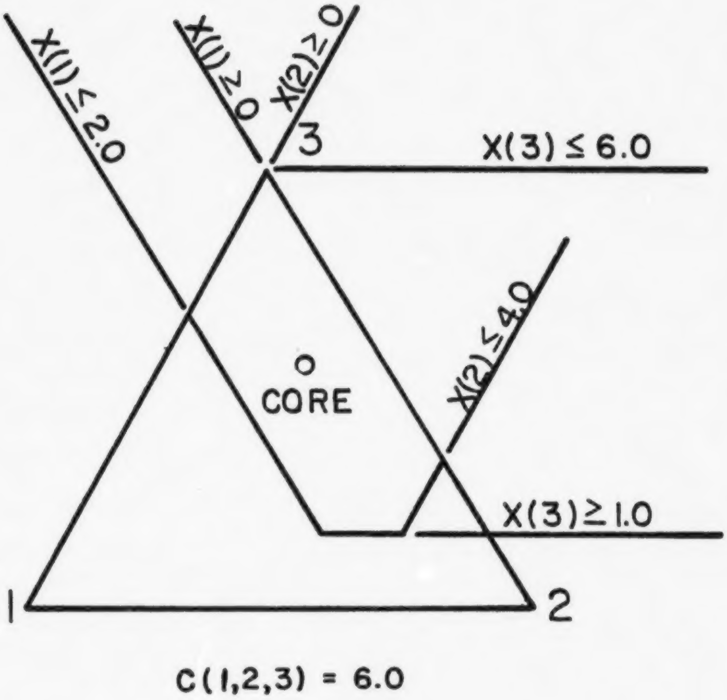
This result is important for several reasons:

1) It unambiguously defines the minimum feasible cost which can be assigned to group i thereby eliminating the problem of what is meant by marginal or incremental costs. Thus, $x(i)_{min}$ should replace SC(i) in the economic criteria.

2) Because it is a linear program, it is very easy to calculate, even for large games, if information on the values for all coalitions are available, or to approximate, using values which are available. The separable costs, remaining benefits method includes only the costs of going alone, C(i), being next to last to join, C(N-{i}), and the cost of the grand coalition. The costs of all other coalitions are not included.

3) Because it is a feasible solution, the calculated value of x(i) can be constrained to be not less than $x(i)_{min}$.

If the game has no core, and problem 6-58 is solved, then $\sum_{i\varepsilon N} x(i) > C(N)$. A way to satisfy $\sum_{i\varepsilon N} x(i) = C(N)$ is to make the conditions on the intermediate coalitions less restrictive. Since it is impossible to satisfy the intermediate constraints, it seems reasonable to relax the constraints on the sub-coalitions until a core appears. If there are any savings then the set of imputations is nonempty. It is the added constraints on the sub-coalitions, S, that eliminate the core. However, a proportionate relaxation of these constraints will yield a nonempty core. Thus, if no core exists, solve the following problem:

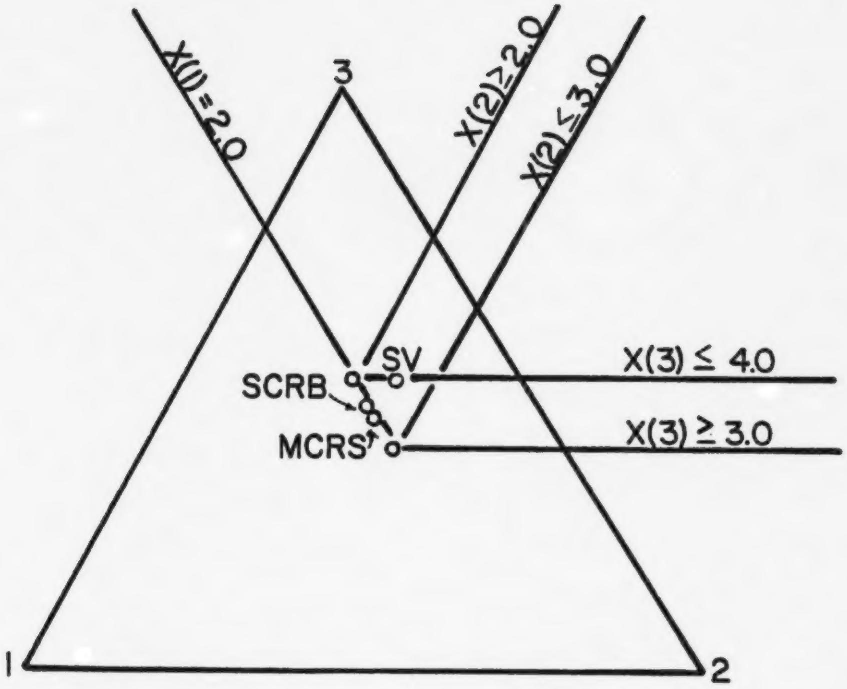

X(1) ≤ 2.0
X(1) ≥ 0
X(2) ≥ 0
3
X(3) ≤ 6.0
X(2) ≤ 4.0
CORE
X(3) ≥ 1.0
1
2
C(1,2,3) = 6.0

SHAPLEY VALUE = [0.83, 1.83, 3.34]

SEPARABLE COSTS,
REMAINING BENEFITS
SOLUTION = [0.91, 1.82, 3.27]

MINIMUM COSTS,
REMAINING SAVINGS
SOLUTION = [0.91, 1.82, 3.27]

Figure 6-7. Convex Cost Game with Core.


X(1)=2.0
3
X(2)≥2.0
X(2)≤3.0
SV
X(3) ≤ 4.0
SCRB
MCRS
X(3) ≥ 3.0
1
2

C(1,2,3) = 8.0

SHAPLEY VALUE = [1.50, 2.50, 4.00]

SEPARABLE COSTS, REMAINING BENEFITS SOLUTION = [2.00, 2.40, 3.60]

MINIMUM COSTS, REMAINING SAVINGS SOLUTION = [2.00, 2.50, 3.50]

Figure 6-8. Nonconvex Cost Game with Core.

$$\text{minimize } \theta$$

$$\begin{aligned}
\text{subject to} \quad & x(i) \leqq [B(i), C(i)] && \forall\ i \varepsilon N \\
& \sum_{i \varepsilon S} x(i) - \theta\{\min[B(S), C(S)]\} \leqq \min\,[B(S), C(S)] && \forall\ S \quad N \qquad (6\text{-}59) \\
& \sum_{i \varepsilon N} x(i) = C(N), \text{ and} \\
& x(i) \geqq 0 && \forall\ i \varepsilon N
\end{aligned}$$

The optimal solution to this problem tells the minimum value, θ^*, which yields a nonempty core. The results of this analysis are inserted into the following formulation to find the minimum cost for group i, i.e.,

$$\text{minimize } x(i)$$

$$\begin{aligned}
\text{subject to} \quad & x(i) \leqq \min\,[B(i), C(i)] && \forall\ i \varepsilon N \\
& \sum_{i \varepsilon S} x(i) \leqq \min[B(S), C(S)]\ (1+\theta^*) && \forall\ S \quad N \qquad (6\text{-}60) \\
& \sum_{i \varepsilon S} x(i) = C(N), \text{ and} \\
& x(i) \geqq 0 && \forall\ i \varepsilon N
\end{aligned}$$

As an example, consider the following subadditive cost game without a core

$$C(1) = 4,\ C(2) = 4,\ C(3) = 5$$

$$C(12) = 5,\ C(13) = 8,\ C(23) = 6$$

$$C(123) = 10$$

These characteristic function values imply the following bounds on the charges to the three groups: $x(1) = 4$, $2 \leqq x(2) \leqq 4$, and $x(3) = 5$. Given unique values of $x(1) = 4$ and $x(3) = 5$, then $x(2) = 1.0$ which is impossible since $2 \leqq x(2) \leqq 4$.

Thus, the values on the two player coalitions are relaxed by solving problem 6-59. For this example, $\theta^* = 0.0526$ and the resulting θ core is the unique point [3.68, 1.58, 4.74]. The Shapley value is [3.33, 2.33, 4.34]. The separable costs, remaining benefits method runs into problems when the game has no core. The separable costs are [4,2,5]. The nonseparable costs are -1. For this case, the nonseparable cost weighting factors, B_i, are not clearly defined since some of the values are zero. The SCRB solution is [4,1,5] which allots all of the savings to Player 2, not a very reasonable compromise solution.

Finding the Maximum Cost Allocation to Group i

In the previous section, it was argued that the minimum allocated cost should be a feasible solution to the game, i.e., within the core. Correspondingly, the maximum allocated cost should also be feasible. From condition (6-35),

$$x(i) \leqq \min[B(i), C(i)] \qquad \forall\ i \varepsilon N \qquad (6\text{-}61)$$

But, for games with cores, is this solution always feasible? The answer is no as shown by the example with C(123) = 8.0. Referring to Figure 6-8, the maximum feasible cost allocations are

$$x_{max} = [2.0, 3.0, 4.0]$$

In this case, $x(i)_{max} \leqq C(i)\ \ \forall(i)$. From before, the minimum values of x(i) are $x_{min} = [2.0, 2.0, 3.0]$. Assume that all other groups j are allotted their minimum cost. How much would group i pay? He pays

$$x(i)_{max} = \min[B(i), C(i), C(N) - \sum_{\substack{j=1 \\ j\neq i}}^{n} x(j)_{min}] \qquad (6\text{-}62)$$

In this case

$$x(1)_{max} = \min[6, 2, 8.0 - 2.0 - 3.0] = 2.0$$

$$x(2)_{max} = \min[7, 4, 8.0 - 2.0 - 3.0] = 3.0$$

$$x(3)_{max} = \min[6, 6, 8.0 - 2.0 - 2.0] = 4.0$$

as shown in Figure 6-8 as the upper bound on the core. In general, the maximum cost can be found by solving the linear program,

$$\begin{aligned} &\text{maximize} \quad x(i) \\ &\text{subject to } x(i) \leqq \min[B(i), C(i)] && \forall\ i\varepsilon N, \\ &\sum_{i\varepsilon S} x(i) \leqq \min[B(S), C(S)](1+\theta^*) && \forall\ S\ \ N \\ &\sum_{i\varepsilon N} x(i) = C(N), \text{ and} && (6\text{-}63) \\ &x(i) \geqq 0 && \forall\ i\varepsilon N. \end{aligned}$$

For games without cores, θ^* is greater than zero.

Solution Concepts

The conventional method uses the separable costs, remaining benefits method to find a "fair" solution. Analogously, the Shapley value and other solution concepts are used in game theory. Referring to the example, the SCRB and Shapley value solutions are shown in Figures 6-7 and 6-8. The two solutions are almost identical for the convex game, C(123) = 6.0. The Shapley value lies outside the core for the nonconvex game, C(123) = 8.0. For the example of a game without a core, the separable costs, remaining benefits method does not work well since some of the remaining benefits are zero.

It would seem reasonable to argue that the fairest solution is the one that falls in the "center" of the core. Unfortunately, the choice of center depends on the selected measure of distance. The distance between two points with coordinate location (x_a, y_a) and (x_b, y_b) can be a

a) Rectilinear measure

$$d_{ab} = x_a - x_b + y_a - y_b \text{ , or} \qquad (6\text{-}64)$$

b) Euclidean measure

$$d_{ab} = [(x_a - x_b)^2 + (y_a - y_b)^2]^{\frac{1}{2}} \text{ or} \qquad (6\text{-}65)$$

c) 1_p norm

$$d_{ab} = [x_a - x_b)^p + (y_a - y_b)^p]^{\frac{1}{p}} \qquad (6\text{-}66)$$

The appropriate measure of distance may depend on the particular problem definition. Fortunately, in most practical applications, the results one obtains using different measures of distance are quite close to one another and well within the accuracy of the cost data used for the problem.

If the game is convex, the Shapley value and the SCRB approximation are acceptable. The unresolved problem occurs when the game is not convex since these methods are now taking weighted averages of infeasible solutions. The proposed remedy outlined below can be expressed as a generalization of the SCRB method.

Step 1. Find the minimum ($x(i)_{min}$) and maximum ($x(i)_{max}$) costs by solving linear programs 6-58 or 6-59, and 6-62 respectively.

Step 2. Find the proration of the nonseparable cost as before, where

$$\beta(i) = \frac{x(i)_{max} - x(i)_{min}}{\sum_{i\varepsilon N}[x(i)_{max} - x(i)_{min}]} \qquad \forall\ i\varepsilon N \qquad (6\text{-}39)$$

$$\text{and NSC} = C(N) - \sum_{i\varepsilon N} x(i)_{min} \qquad (6\text{-}38)$$

Step 3. Find the fair solution for each group using

$$x(i) = x(i)_{min} + \beta(i)\ (NSC) \qquad (6\text{-}67)$$

For lack of a better name, this method is called the minimum cost, remaining savings method (MCRS).

CONCLUSIONS

Procedures for efficiency/equity analysis have evolved over the past 40 - 50 years in the water resources field. The specific motivation for this study is to recommend procedures for environmental analysis to assist the U.S. Environmental Protection Agency in managing a $\$5 \times 10^9$/yr water pollution control effort.

In the water resources field, intensive studies of the cost allocation problem have been made in response to controversy surrounding the early development in the 1930's of the Tennessee Valley Authority in the Southeastern U.S. The growth of federal involvement in U.S. water projects during the 1930's and 1940's stimulated the development of generalized procedures for efficiency/equity analysis. These methods are still widely used. These efficiency/equity criteria are used to evaluate the suitability of the proposed procedures.

Related criteria were developed by game theorists over a similar time span. Of specific relevance is core theory and solution notions such as the Shapley value. During the past several years, investigators have attempted to demonstrate the relevance of these methods to water resources and environmental problems. Remarkably, the TVA experts (3) had stated some of the key assumptions of core theory as related to the TVA cost allocation problem. Unfortunately, no subsequent work was done on this subject. One correction in the conventional theory is needed to have conditions equivalent to the core. The characteristic function for an s member coalition needs to be defined as the minimum of the benefits and costs since it is possible for subcoalitions to not satisfy the economic efficiency criterion that $B(S) \geq C(S)$. With that change, the two approaches are equivalent.

The next major task was to determine the minimum costs which a purpose must be assessed. Separable costs are defined as the incremental costs of adding a group as the last group or

$$SC(i) = C(N) - C(N - \{i\}) \qquad \forall\ i \in N \tag{6-67}$$

It has been assumed that this is the minimum cost which group i could be assessed and is thereby the relevant one to compare to its marginal benefits. Another way of defining these minimum costs is to determine the direct costs, i.e., the costs of those project elements serving only one purpose.

The direct cost method is shown to be incorrect since it fails to recognize the net change in total cost that each project element causes. It is the total change in cost which is relevant.

Separable costs, as defined, are minimum incremental costs only if the game is convex; often it is not. When the game is nonconvex a group could have a lower incremental cost if it joined earlier. Thus, accepted practices for economic/equity analysis are incorrect when the game is nonconvex.

A general method for finding the minimum costs for purpose i is to solve the linear program which minimizes $x(i)$ subject to the core constraints.

This method is conceptually sound and computationally simple since efficient computer codes are widely available. An exact solution can be found or an approximation obtained using available characteristic function data. If the game does not have a core, the inequalities on the s member coalition constraints are relaxed and the minimum costs for that problem are found. Analogous procedures are used to find the maximum feasible costs.

Knowing the minimum and maximum feasible costs for each player, a generalization of the separable costs, remaining benefits method is proposed as the solution procedure. Each purpose is assigned its minimum costs (instead of its separable costs) and a proportionate share of the remaining costs based on the difference between its minimum and maximum assignable costs. Maximum feasible costs are used instead of benefits. The resulting formula is identical to accepted practice if $x(i)_{min}$ and $x(i)_{max}$ are used in place of separable costs and benefits. The solution of this game is guaranteed to be in the core since it is a convex combination of two feasible solutions. Also, because it is directly related to accepted practice, it should be much easier to implement than solutions which are unfamiliar to practitioners.

The minimum cost, remaining benefits (MCRS) method is preferable to the accepted separable costs, remaining benefits (SCMB) method in several respects. First, the minimum cost which is assigned to each group is guaranteed to retain stable coalitions if a core exists. On the other hand, the separable costs might not be feasible in the sense that such an allocation would leave all other groups satisfied in that it is the core. Another reason for preferring the MCRS method is that it uses all of the available coalition information when there are more than three groups. The SCRB method only examines coalitions of size of 1, n-1, and n. By using maximum feasible savings as an upper bound the MCRS method assures that joint costs are allocated in proportion to ability to pay as measured by the difference between the minimum and maximum charges which can be assessed on any group.

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CHAPTER 7

CASE STUDIES IN ECONOMIC/FINANCIAL ANALYSIS

INTRODUCTION

The previous chapter describes some new concepts which may be helpful in examining the economic and financial aspects of environmental problems. As the analysis indicated, there is no single best way to approach these problems. However, sound concepts do exist to permit a more systematic evaluation, especially of the financial aspects of problems. A rich body of economic theory exists to help evaluate this aspect of the problem. Unfortunately, gaps remain in attempting to define the correct notion of costs to use. Economists insist on using marginal cost analysis which often leads to a significant portion of the costs remaining unallocated. How should the practitioner handle such problems? On the other hand, the rules for financial analysis have emerged based on trial and error procedures without the firm support of a unified theoretical foundation. Cooperative N-person game theory appears to offer such a foundation. Indeed, the previous chapter illustrated the remarkable similarity of the two approaches.

Ultimately, practicing professionals will determine the usefulness of the various approaches. The remainder of this chapter presents eight cost allocation/sharing examples to permit the reader to gain a better hands on feel for the procedures.

Case Studies

MCKEAN'S ARGUMENT AGAINST COST ALLOCATION

One of the most widely quoted books on benefit-cost analysis is McKean's, Efficiency in Government Through Systems Analysis (1). He presents the following example (pp. 44-46) which argues against cost allocation.

> "Consider for a moment a multiple-purpose dam of a particular size. Suppose its full cost is $100 million; the cost of the extra capacity and features that are included for power generation amount to $30 million; the costs due solely to irrigation (consisting again of extra capacity and facilities) amount to $30 million. The two $30 million dollar amounts will be called "separable costs"; if either the power or the irrigation functions were dropped from the plan, $30 million would be lopped off the project's cost. The remainder, $40 million, will be called 'joint cost'... Suppose the $40 million in the case of the $100 million multi-purpose dam was allocated by means of some formula. Assume that the formula

> bestowed half of the $40 million for each purpose so that power now appeared to cost $50 million and irrigation $50 million. If the value of benefits from irrigation were $70 million and that from power $40 million (a total of $110 million), should we declare the power facilities unjustified or declare the entire project unjustified? Surely we could only say that the present value of the total benefits should be not less than $100 million, the present value of benefits from power output not less than $30 million, and that from irrigation not less than $30 million. An entrepreneur launching a multi-product firm would presumably think along these lines. Also, economic theory indicates that the costs of multiple products ought to be treated in this manner."

McKean's example does illustrate the important point that inappropriate cost allocation formulas can distort the economic analysis. In this case the joint cost weighting factors, the β_i, were not related to ability to pay which we have already argued is the preferred way.

The proper characteristic function definition is

$$c(S) = \min. \{B(S), C(S)\} \tag{7-1}$$

where

$c(S)$ = maximum cost for coalition S

$B(S)$ = benefits for coalition S

$C(S)$ = cost for coalition S

Then, the three characteristic function values are $c(P) = 40$, $c(I) = 70$, and $c(PI) = 100$. Using the MCRS method yields allocated costs of $x(P) = 38$, and $x(I) = 62$ -- an acceptable solution.

JAMES AND LEE'S THREE PURPOSE EXAMPLE

James and Lee present the following example to illustrate various cost allocation methods (2):

> The Orchard and the Grainlands Irrigation Districts joined forces to construct a water supply reservoir. By enlarging the reservoir, the two districts were also able to provide flood control for the downstream town of Centerville, having an average annual benefit of $400,000. The Orchard Irrigation District requires 50,000 acre-ft (6.17×10^7 m^3) of water per year and receives a unit benefit of $15/acre-ft ($.012/m^3). The Grainlands Irrigation District requires 200,000 acre-ft (2.47×10^8 m^3) of water per year but, because of the lower-value crops grown, receives a unit benefit of only $2/acre -ft ($.0016/m^3). The project has a first cost of $15 million which must be divided among the two irrigation districts and the Federal government, which will pay for

flood control. Of the project first cost, 80 percent is for the reservoir and 20 percent is for a canal from the reservoir to the point where the water going to the two districts separates. The unit benefit figures given above for irrigation water are net of district distribution and on-farm costs.

The costs are allocated in the following steps:

1. The three cost centers are the Orchard Irrigation District, the Grainlands Irrigation District, and the Federal government.

2. Alternative designs reveal the reservoir to cost $10 million if it provides only irrigation water, $10,500,000 if no water is delivered to the Orchard Irrigation District and $8,500,000 if no water is delivered to the Grainlands Irrigation District. Alternative designs reveal that a canal serving only the Orchard Irrigation District would cost $1,500,000 and that a canal serving only the Grainlands Irrigation District would cost $2,500,000.

3. The present worth of the benefits is found by multiplying the average annual value times the uniform-annual-series present-worth factor.

For i = 3 percent, N = 50 years, (P/A, i%, N) = 25.73.

	OID	GID	Flood control
Annual benefit:	$ 750,000	$ 400,000	$ 400,000
Capitalized benefit:	19,275,000	10,280,000	10,280,000

4. Costs of alternative projects serving each center are determined. Flood control can be provided by channel improvement costing $7 million. The Orchard Irrigation District can be served by water from a different river at a cost of $9,500,000. The Grainlands Irrigation District can be supplied for $20 million.

This example illustrates three main points: (1) the importance of using the minimum of the benefits or costs in defining the characteristic function; (2) the use of a different definition of separable costs; and (3) problems with prorating costs in proportion to use.

Let the three cost centers be denoted as

Orchard Irrigation District → 1

Grainlands Irrigation District → 2

Centerville Flood Control → 3

A schematic diagram of the project is shown in Figure 7-1.

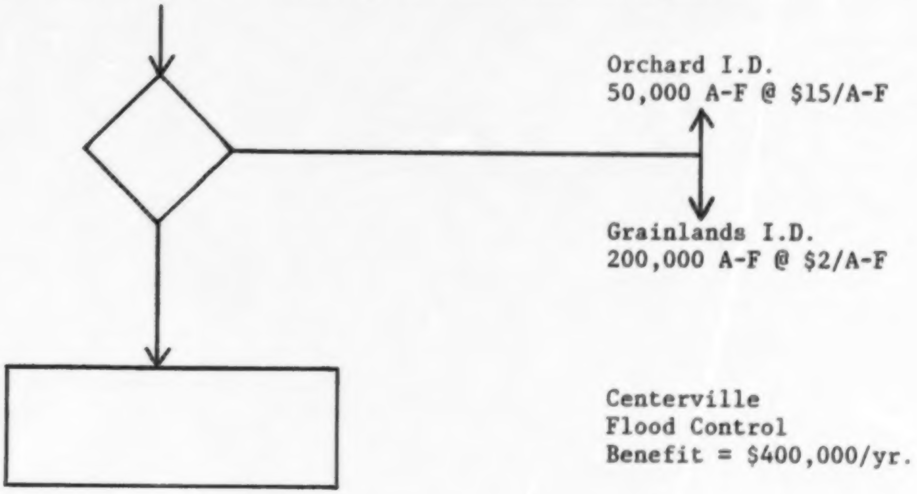
Orchard I.D.
50,000 A-F @ $15/A-F
Grainlands I.D.
200,000 A-F @ $2/A-F
Centerville
Flood Control
Benefit = $400,000/yr.

Figure 7-1. Schematic Diagram of Case Study 2 (2).

A. Find alternative costs for all coalitions

$C(1) = 9.5$, $C(2) = 20.0$, $C(3) = 7.0$

$C(12) = 13.0$, $C(13) = 10.0$, $C(23) = 13.0$

$C(123) = 15.0$

B. Find benefits

$B(1) = 19.3$, $B(2) = 10.3$, $B(3) = 10.3$

C. Find the characteristic function values, $v(S)$

where $v(S) = \min [B(S), C(S)]$

$v(1) = 9.5$, $v(2) = 10.3$, $v(3) = 7.0$

$v(12) = 13.0$, $v(13) = 10.0$, $v(23) = 13.0$

$v(123) = 15.0$

Note that Grainlands Irrigation District would not build a project by itself since $B(2) < C(2)$.

D. Find minimum costs

1. Separable costs taken element by element. James and Lee determine the separable costs element by element as shown below:

	Separable to 1	Separable to 2	Separable to 3	Nonseparable
Reservoir	1.50	3.50	2.0	5.00
Canal	0.50	1.50	0	1.00
Total	2.00	5.00	2.00	6.00

Defining separable costs as C(N) - C(N - [i]) yields

$$SC(1) = c(123) - c(23) = 15.0 - 13.0 = 2.0$$

$$SC(2) = c(123) - c(13) = 15.0 - 10.0 = 5.0$$

$$SC(3) = c(123) - c(12) = 15.0 - 13.0 = 2.0$$

Thus, the results are identical as they should be. It is unnecessary to do an element by element accounting.

2. Minimum Costs

Figure 7-2 shows the core of this game. The minimum costs equal separable costs, or

$$\bar{x}_{min} = [2.0, 5.0, 2.0]$$

E. Find Maximum Costs

From Figure 7-2, the maximum costs are

$$\bar{x}_{max} = [8.0, 10.3, 7.0]$$

Note that the upper bound on X(1) is 8.0, not 9.5.

F. Find Remaining Costs

$$\text{Remaining Costs (RC)} = C(N) - \sum_{i=1}^{3} x(i)_{min} \tag{7-2}$$

$$= 15 - 9$$

$$= 6.0$$

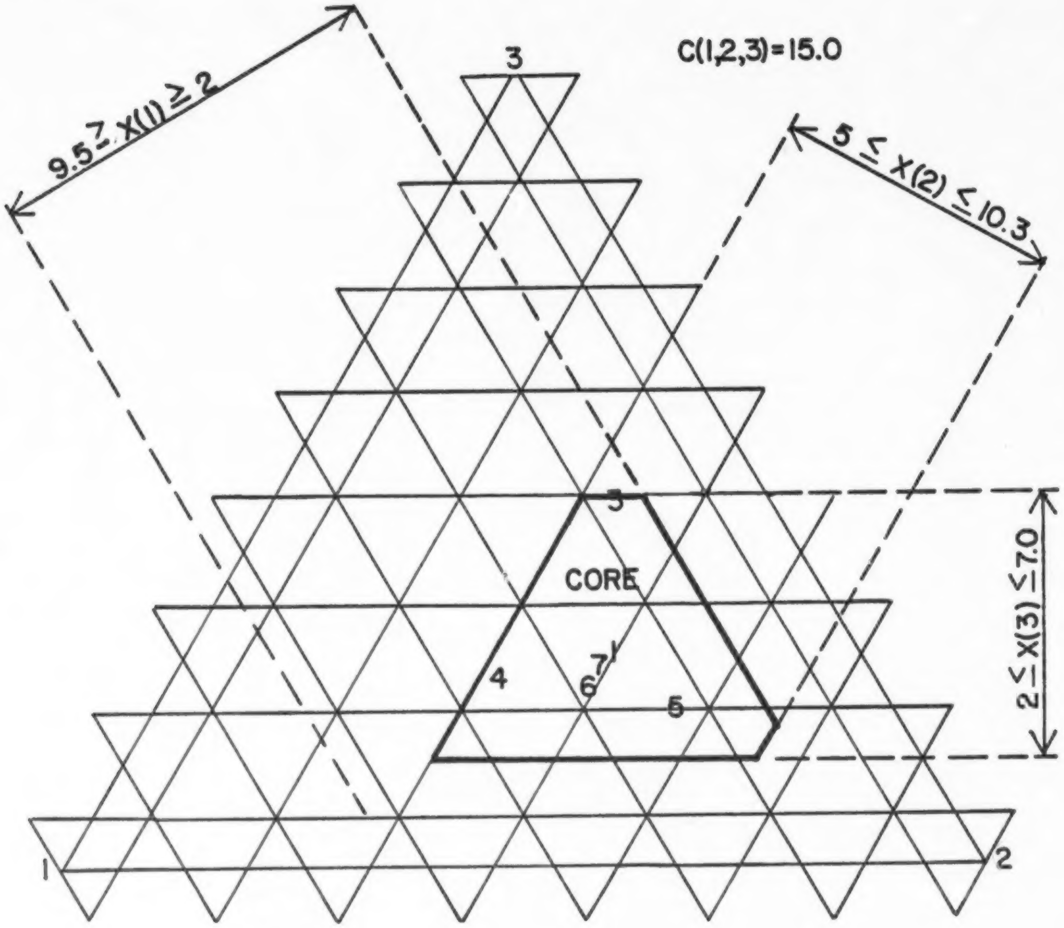
9.5 ≥ X(1) ≥ 2
3
C(1,2,3)=15.0
5 ≤ X(2) ≤ 10.3
3
CORE
7
4
6
5
2 ≤ X(3) ≤ 7.0
1
2

Figure 7-2. Core of Game for Case Study 2.

G. Assign total costs using various weighting factors

$$x(i) = x(i)_{min} + \beta_i\ (RC) \qquad (7\text{-}3)$$

1. β_i is equal for all i implies $\beta_i = 1/3$

$x(1) = 2.0 + 1/3(6) = 4.0$

$x(2) = 5.0 + 1/3(6) = 7.0$

$x(3) = 2.0 + 1/3(6) = \underline{4.0}$

15.0

2. β_i is proportional to use

The two irrigation districts use the water directly. However, the flood control purpose is an indirect use only. Its use relates more to setting aside storage capacity which it will use occasionally. Thus, there is no obvious single measure of use in this example.

3. Entirely to highest priority center up to the limit of his benefit.

Assume flood control is highest priority. Using equation 6-12, and noting that $x(3)_{max} = 7.0$

$7.0 = 2.0 + \beta_3(6)$, or

$\beta_3 = 5/6$

Assume that 1 and 2 have an equal β, then $\beta_1 = \beta_2 = 1/12$ and

$X = [2.5, 5.5, 7.0]$

4. Proportional to excess benefit, EB(i)

$EB(1) = B(1) - x(1)_{min} = 19.3 - 2.0 = 17.3$

$EB(2) = B(2) - x(2)_{min} = 10.3 - 5.0 = 5.3$

$EB(3) = B(3) - x(3)_{min} = 10.3 - 2.0 = \underline{8.3}$

30.9

$\beta = [.56, .17, .27]$, and

$X = [5.36. 6.02, 3.62]$

5. Proportional to excess cost of next best alternative, EC(i)

$EC(1) = C(1) - x(1)_{min} = 9.5 - 2. = 7.5$

$EC(2) = C(2) - x(2)_{min} = 20.0 - 5. = 15.0$

$EC(3) = C(3) - x(3)_{min} = 7.0 - 2. = \underline{5.0}$

27.5

$\beta = [.27, .55, .18]$, and

$X = [3.62, 8.30, 3.08]$.

6. Proportional to minimum of excess benefits or excess costs EBC(i)

$EBC(1) = \min[B(1), C(1)] - x(1)_{min} = 7.5$

$EBC(2) = \min[B(2), C(2)] - x(2)_{min} = 5.3$

$EBC(3) = \min[B(3), C(3)] - x(3)_{min} = \underline{5.0}$

17.8

$\beta = [.42, .30, .28]$, and

$X = [4.52, 6.8, 3.68]$.

7. Proportional to difference between $x(i)_{max}$ and $x(i)_{min}$, $\Delta X(i)$

$\Delta X(1) = x(1)_{max} - x(1)_{min} = 8.0 - 2.0 = 6.0$

$\Delta X(2) = x(2)_{max} - x(2)_{min} = 10.3 - 5.0 = 5.3$

$\Delta X(3) = x(3)_{max} - x(3)_{min} = 7.0 - 2.0 = \underline{5.0}$

16.3

$\beta = [.37, .32, .31]$, and

$X = [4.22, 6.92, 3.86]$.

The results of the six different cost allocation methods are compared in Figure 7-2. Three of the solutions (1, 6, 7) fall near the center of the core. The priority of use solution, number 3, lies on the edge of the core since no excess benefits are alloted to player 3. Solutions 4 and 5 are not too close to the center of the core either.

LOUGHLIN'S WATER RESOURCES EXAMPLE

Loughlin presents a current summary of efficiency/equity analysis in the water resources field (3). He discusses and restates the Green Book's three criteria for a project to be economically justifiable and properly formulated:

1) The total of all benefits exceeds the total of all costs.
2) The scale of development is such as to provide maximum total benefits over total costs.
3) The benefits of adding each purpose as the last increment at least equal the separable costs of the added capacity.

Loughlin's interpretation of these three criteria is presented below:

> For project feasibility the overall benefit-cost ratio must be greater than unity. This involves the comparison of totals (benefits and costs), which precludes the necessity to allocate joint costs among purposes. In deciding the optimal design of a project, marginal or incremental analysis is employed to determine that scale of development where marginal social benefits are equal to marginal social costs. As long as marginal benefits exceed marginal costs, an increase in the scale of development will increase net benefits. Since marginal costs are independent of joint costs, a cost allocation is not necessary to determine optimal scale. Similarly, in deciding whether to include a function in the project an incremental analysis is employed. As long as the additional benefits from including the function are equal to or greater than the additional or separable costs of including the function, inclusion of the purpose is justified. The benefits of each purpose need only cover separable costs, since joint costs are sunk and need not be recouped. This eliminates the necessity for purpose benefits to cover a part of joint costs and therefore the need to allocate joint costs among functions in deciding purpose justification.

This line of reasoning has caused much confusion because it concludes that joint costs need not be recouped which is unacceptable accounting practice. The game theoretic approach can shed some light on this apparent anomoly. Loughlin's four purpose example, shown in Table 7-1, is examined below:

A. Notation

Let purpose A be denoted by 1
purpose B be denoted by 2
purpose C be denoted by 3
purpose D be denoted by 4

B. Find Benefits. From Table 7-1,

$B(1) = 140, B(2) = 100, B(3) = 150, B(4) = 100$

C. Find costs for all coalitions. From Table 7-1, for independent action, alternative costs are used. Thus, $C(1) = 100$, $C(2) = 120$, $C(3) = 100$, $C(4) = 130$. Costs for the pairs are not included. They will be estimated later. The costs for any triplet can be calculated since separable costs are given. Since

$$SC(i) = C(N) - C(N-[i]), \text{ or}$$
$$C(N-[i]) = C(N) - SC(i)$$
$$C(123) = C(1234) - SC(4) = 350 - 0 = 350$$
$$C(124) = C(1234) - SC(3) = 350 - 20 = 330$$

Item	Description	Group				
		A	B	C	D	Total
1	Benefits	140	100	150	100	490
2	Alternative costs	100	120	100	130	450
3	Justifiable costs (lessor of 1 or 2)	100	100	100	100	400
4	Separable costs	80	50	20	0	150
5	Remaining benefits (3 - 4)	20	50	80	100	250
6	Allocated joint costs[a]	16	40	64	80	200
7	Total allocated costs (4 + 6)	96	90	84	80	350
8	Allocated savings (3 - 7)	4	10	16	20	50
9	Percent allocated savings	8	20	32	40	100
10	Cost of project combining all other purposes (project cost - 4)	270	300	330	350	...
11	Justifiable costs combining all other purposes[b]	270	300	300	300	...
12	Savings from purpose inclusion (3 + 11 - project cost)	20	50	50	50	170
13	Percent savings from purpose inclusion	12	29	29	29	99[c]

Values are in thousands of dollars except where it is otherwise indicated.

[a] Total joint costs of $200 (project cost of $350 minus total separable cost of $150) are allocated to each purpose in the same ratio as that of the remaining benefits of each purpose to the total remaining benefits. For purpose A this becomes ($20/$250) x $200 = $16.

[b] The justifiable costs combining all other purposes are either the cost of a project combining all other purposes (row 10) or the sum of the justifiable single-purpose costs for all other purposes (row 3), whichever is the lesser.

[c] Totals do not equal 100% owing to rounding.

Table 7-1. SCRB Method of Cost Allocation (3)

$C(134) = C(1234) - SC(2) = 350 - 50 = 300$
$C(234) = C(1234) - SC(1) = 350 - 80 = 270$

Check For Consistency

a) $C(123) = 350 \leq C(1) + C(2) + C(3) = 320$. No.
$C(123)$ is inefficient as defined. Correct $C(123) = 320$

b) $C(124) = 330 \leq C(1) + C(2) + C(4) = 350$ OK

c) $C(134) = 300 \leq C(1) + C(3) + C(4) = 330$ OK

d) $C(234) = 270 \leq C(2) + C(3) + C(4) = 350$ OK

Thus, the corrected values of the triplets are $C(123) = 320$, $C(124) = 330$, $C(134) = 300$, $C(234) = 270$.

The cost for the pairs will be assumed. To be consistent.

a) $C(12) \leq C(1) + C(2) = 220$
$C(12) \geq C(123) - C(3) = 220$
$C(12) \geq C(124) - C(4) = 200$
or $220 \leq C(12) \leq 220$
$C(12) = 220$

b) $C(13) \leq C(1) + C(3) = 200$
$C(13) \geq C(123) - C(2) = 200$
$C(13) \geq C(134) - C(4) = 200$
or $200 \leq C(13) \leq 200$
$C(13) = 200$

c) $C(14) \leq C(1) + C(4) = 230$
$C(14) \geq C(124) - C(2) = 210$
$C(14) \geq C(134) - C(3) = 200$
or $210 \leq C(14) \leq 230$
Let $C(14) = 220$

d) $C(23) \leq C(2) + C(3) = 220$
$C(23) \geq C(123) - C(1) = 220$
$C(23) \geq C(234) - C(4) = 140$
or $220 \leq C(23) \leq 220$
$C(23) = 220$

e) $C(24) \leq C(2) + C(4) = 250$
$C(24) \geq C(234) - C(3) = 170$
$C(24) \geq C(124) - C(1) = 230$
or $230 \leq C(24) \leq 250$
Let $C(24) = 240$

f) $C(34) \leq C(3) + C(4) = 230$
$C(34) \geq C(134) - C(1) = 200$
$C(34) \geq C(234) - C(2) = 150$

or $200 \leqq C(34) \leqq 230$
Let $C(34) = 200$

D. Find the characteristic function, $c(S)$

Where $c(S) = \min [B(S), C(S)]$
$c(1) = 100$, $c(2) = 100$, $C(3) = 100$, $c(4) = 100$
$c(12) = 220$, $c(13) = 200$, $c(14) = 220$, $c(23) = 220$, $c(24) = 240$
$c(34) = 200$
$c(123) = 320$, $c(124) = 330$, $c(134) = 300$, $c(234) = 270$, $c(1234) = 350$

E. Find the incremental costs for each group joining 1, 2, 3, or 4.

Table 7-2 shows the results for $c(1234) = 350$ and 365. If $c(1234) = 350$, then the lowest incremental cost for every group results when he joins last. However, this is not the case if $c(1234) = 365$. Now it is better for groups 1 and 3 to join third since incremental costs are minimum for that ordering. Thus, the economic criterion of checking marginal benefits vs. separable costs may lead to erroneous conclusions where separable costs are not the minimum incremental costs. This is a very serious shortcoming.

Loughlin is correct that truly joint costs are sunk and need not be recouped. However, only in one of seven cost allocation procedures are such costs sunk, i.e., when the purposes agree a priori on the order in which they will join. This results in a disproportionate amount of the savings accruing to the purposes which join at or near the end. But this is hardly a typical bargaining situation. All groups have a right to a share of the savings and could be expected to bargain accordingly. Then the joint costs would be zero and all costs would be variable. Alternatively, the incremental costs defined as $C(N) - C(N - [i])$ are not jointly achievable. Thus, they should not be viewed as correctly representing the actual costs of adding a purpose as the last one. Rather, they are the incremental costs, only for the exceptional case, where the other N-1 players naively agree to underwrite all of the nonseparable costs. Thus, joint costs do not typically exist because the players are still bargaining and they would be foolish to commit themselves too early in the negotiations.

The general intent of the principle in the Green Book is correct but the problem needs to be examined more carefully than they outline. Indeed, in many cases, it may be necessary to iterate back and forth between the economic optimization, with provisional incremental cost data, and the cost allocation problem.

FOUR PURPOSE EXAMPLE FROM THE GREEN BOOK

The report to the Inter-Agency Committee on Water Resources, commonly called the Green Book, contains a four group cost game (4). The players are: (1) Flood Control, (2) Power, (3) Irrigation, and (4) Navigation. The costs are allocated by the Separable Cost-Remaining Benefits method (SCRB).

Incremental Cost of Joining Indicated Coalition Last

	Group			Group			Group	Group	Total
I	2	3	4						
1	100		---	---	---				
2	---	100	---	---					
3	---	---	100	---					
4	---	---	---	100(I)					
12	120(I)		120(I)		---	---			
13	100	---	100	---					
14	120(I)		---	---	120(I)				
23	---	120(I)		120(I)		---			
24	---	140(I)		---	140(I)				
34	---	---	100	100					
123	100	120(I)		100	---				
124	90	110(I)		---	110(I)				
134	100	---	80	100					
234	---	70	30	50					
1234	80	50	20	20			c(1234) = 350		
(1234)'	95	65	35	35			c(1234) = 365		

I → Inefficient Incremental Cost

Table 7-2. Incremental Costs of Joining Coalitions in Different Orders

$B(1) = 500$, $B(2) = 1500$, $B(3) = 350$, $B(4) = 100$
$C(1) = 400$, $C(2) = 1000$, $C(3) = 600$, $C(4) = 80$
$C(123) = 1715$, $C(124) = 1615$, $C(134) = 1165$
$C(234) = 1385$, $C(1234) = 1765$

Table 7-3. Benefits and Costs from Green Book

Data for this example, as presented in the Green Book, are shown in Table 7-3. The purpose of presenting this example is to illustrate the importance of checking that the characteristic function for each coalition is calculated correctly. Recall condition 6-45 which defines

$$c(S) = \text{Min}\ [B(S), C(S)] \tag{7-4}$$

This characteristic function value gives the minimum alternative justifiable expenditure for coalition S. The benefits to S are simply the sum of the benefits for the members. However, the costs are the minimum cost solution for coalition S. Using condition (7-4) to calculate the characteristic function values yields (all costs or benefits in thousand dollars)

$$C(1) = 400,\ C(2) = 1000,\ C(3) = 350,\ C(4) = 80$$

$$C(123) = 1715,\ C(124) = 1480,\ C(600) = 1385,\ C(134) = 950,$$

$$C(1234) = 1765$$

Table 7-4. Corrected Benefits and Costs for Green Book Example.

Unfortunately, the Green Book uses two economically irrational coalitions, 124 and 134. An irrational coalition in a cost game is a coalition in which the members of the coalition can save money by leaving the coalition and forming sub-coalitions. For example, it is necessary that

$$C(1) + C(2) + C(4) \geqq C(124) \text{ for } C(124)$$

to be an essential coalition. But

$$400 + 1000 + 80 = 1480 < 1615.$$

Thus, the reported value of coalition 124 is incorrect. The correct characteristic function cost of 124 should be 1480, not 1615.

Similarly, $C(1) + C(3) + C(4) \geqq C(134)$ for $C(134)$

to be an essential coalition. But

$$400 + 600 + 80 = 1080 < 1165.$$

Thus, coalition 134 is irrational. Furthermore, the possible benefit to coalition 134 is only 500 + 350 + 100 = 950. The correct characteristic function for 134 is not 1165 or even 1080 but 950. A coalition cannot be charged more than the minimum of its benefits or costs or the resulting cost allocation may not satisfy the core imputations.

The correct characteristic functions benefit or cost for all given coalitions are shown below.

Single group coalitions

$c(1) = 400$ $c(3) = 350$
$c(2) = 1000$ $c(4) = 80$

Three group coalitions

$c(123) = 1715$ $c(134) = 950$
$c(124) = 1480$ $c(234) = 1385$

Four group coalition

$c(1234) = 1765$

The bounds of the SCRB method are the costs of independent action and the separable costs. They are

$$\begin{aligned} 380 &\leqq x(1) \leqq 400, \\ 600 &\leqq x(2) \leqq 1000, \\ 150 &\leqq x(3) \leqq 350, \text{ and} \\ 50 &\leqq x(4) \leqq 80. \end{aligned}$$

Solving the linear program with eight different objective functions yields the MCRS method's upper and lower bounds, i.e.,

$$\begin{aligned} 380 &\leqq x(1) \leqq 400, \\ 935 &\leqq x(2) \leqq 1000, \\ 285 &\leqq x(3) \leqq 350, \text{ and} \\ 50 &\leqq x(4) \leqq 80. \end{aligned}$$

The difference between the MCRS and SCRB methods lies in the lower bounds of groups (2) and (3). Group (2) cannot be charged less than 935 since the most groups (1), (3), and (4) can be charged is 400 + 350 + 80 = 830. Thus, group (2) has to be charged at least 1765 - 830 = 935.

The characteristic function cost of coalition 124 was found to be inessential. The maximum cost of 124 was lower thereby raising the minimum or separable cost of player (3).

Separable cost of 3 = 1765 - 1480 = 285. The MCRS and SCRB costs are given below.

Group	MCRS Cost	SCRB Cost
(1)	392.78	398
(2)	976.53	960
(3)	326.53	330
(4)	69.16	77

The SCRB allocated costs to groups (1) and (4) are very close to the maximum cost allowable to groups (1) and (4), respectively 400 and 80. Groups (1) and (4) have only a small incentive to participate in the grand coalition of 4 groups. The MCRS, by allocating smaller costs to groups (1) and 4), increases the stability of the grand coalition.

FOUR CITY SEWAGE TREATMENT SYSTEM

Four cities are considering a cooperative venture to reduce sewage treatment costs (see Figure 7-3). The cost of piping is shown on the figure. The cost for treating various rates of flow 730,000 $Q^{.716}$ where Q = flow rate, cfs.

a) Determine the optimal solution total for all groupings of the four cities, e.g., C(124), C(23).

b) Allocate costs by the use of facilities method wherein each group pays in proportion to the flow he contributes to each component (pipeline, treatment plant).

c) Allocate costs using the Shapley value approach;

d) List the constraints specifying the core. Does the Shapley Value fall in the core?

Solution to part (a)

All C(S) are expressed in $1000's.

C(1) = 1970 - only sewage treatment plant costs
C(2) = 2300 - only sewage treatment plant costs
C(3) = 1600 - only sewage treatment plant costs
C(4) = 2300 - only sewage treatment plant costs

Let C'(12) = cost of piping waste from City 1 to 2 plus the cost of treating the waste streams 1 and 2 at City 2's plant

Then C'(12) = 290 + 3500 = 3790

Similarly
C'(13) = 290 + 230 + 2950 = 3470
C'(14) = 290 + 230 + 403 + 3500 = 4423
C'(23) = 300 + 3200 = 3500
C'(24) = 300 + 525 + 3800 = 4625
C'(34) = 403 + 3200 = 3603

Let C'(123) = least cost of piping 1 and 2 to 3 and treating wastes from 1,2, and 3 @ City 3 plant

Then C'(123) = 290 + 300 + 4300 = 4890

Similarly
C'(124) = 290 + 300 + 630 + 4800 = 6020
C'(134) = 290 + 230 + 525 + 4300 = 5345
C'(234) = 300 + 630 + 4600 = 5530

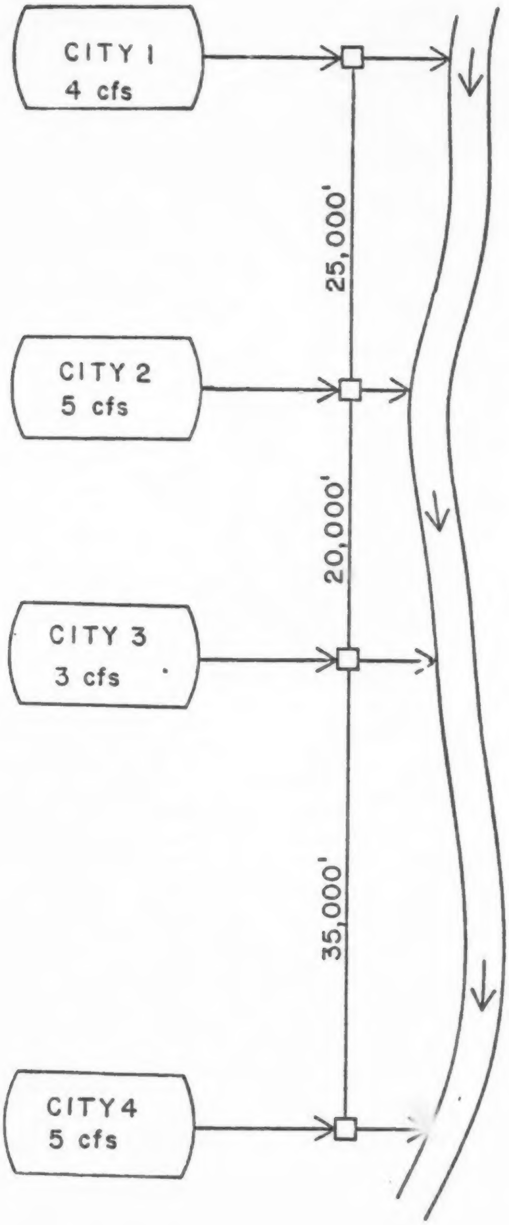
CITY 1
4 cfs
25,000'
CITY 2
5 cfs
20,000'
CITY 3
3 cfs
35,000'
CITY4
5 cfs

Figure 7-3. Arrangement of Four City Sewage Treatment System.

Let C'(1234) = least cost of piping 1, 2, and 3 to 4 and treating wastes of 1, 2, 3 and 4 @ 4.

Then C'(1234) = 290 + 300 + 840 + 5500 = 6930

Independent and Coalition Groupings

C(1) + C(2) = 4270
C'(12) = 3790 Optimal

C(1) + C(3) = 3570
C'(13) = 3470 Optimal

* C(1) + C(4) = 4270 Optimal
C'(14) = 4423

C(2) + C(3) = 3900
C'(23) = 3500 Optimal

* C(2) + C(4) = 4600 Optimal
C'(24) = 4625

C(3) + C(4) = 3900
C'(34) = 3603 Optimal

C(1) + C(2) + C(3) = 5870
C(1) + C'(23) = 5470
C'(12) + C(3) = 5390
C'(13) + C(2) = 5770
C'(123) = 4890 Optimal

C(1) + C(2) + C(4) = 6570
C(1) + C'(24) = 6595
C(2) = C'(14) = 6723
C'(12) + C(4) = 6090
C'(124) = 6020 Optimal
C(1) + C(3) + C(4) = 5870
C(1) + C'(34) = 5573
C'(13) + C(4) = 5770
C'(14) + C(3) = 6023
C'(134) = 5345 Optimal

C(2) + C(3) + C(4) = 6200
C(2) + C'(34) = 6203
C'(23) + C(4) = 5800
C'(24) + C(3) = 6225
C'(234) = 5530 Optimal

C(1) + C(2) + C(3) + C(4) = 8170
C(1) + C(234) = 7500
C(2) + C(134) = 7645
C(3) + C(124) = 7720

$C(4) + C(123) = 7190$
$C(12) + C(34) = 7395$
$C(13) + C(24) = 8095$
$C(23) + C(14) = 7720$
$C(1234) = 6930$ Optimal

Optimal Groupings - SUMMARY

(Combination of Independents or Coalitions)
\$ x 1000

$c(1) = 1970$
$c(2) = 2300$
$c(3) = 1600$
$c(4) = 2300$

$c(123) = 4890$
$c(124) = 6020$
$c(134) = 5345$
$c(234) = 5530$

$c(12) = 3790$
$c(13) = 3470$
$c(14) = 4270$
$c(23) = 3500$

$c(1234) = 6930$

$c(24) = 4600$
$c(34) = 3603$

b) Use of Facilities Method for Cost Sharing (All values in \$ x 1000)

The user cost is based on the flow fraction of each component (piping, treatment).

Grand Coalition

$$C(1234) = \$6{,}930$$

City 1

$$C_1 = 1.0\,(290) + \frac{4}{9}\,(300) + \frac{4}{12}\,(840)$$
$$+ \frac{4}{17}\,(5{,}500) = \$1{,}997$$

$$C_1 = 1{,}997 \text{ vs. } C(1) = \$1{,}970 \text{ acting independently}$$

City 2

$$C_2 = \frac{5}{9}\,(300) + \frac{5}{12}\,(840) + \frac{5}{17}\,(5{,}500)$$
$$= \$2{,}134$$

$$C_2 = \$2{,}134 \text{ vs. } C(2) = \$2{,}300 \text{ acting independently}$$

City 3

$$C_3 = \frac{3}{12}\,(840) + \frac{3}{17}\,(5{,}500)$$
$$= \$1{,}181$$

C_3 = \$1,181 vs. C(3) = \$1,600 acting independently

City 4

$$C_4 = \frac{5}{17}(5{,}500) = \$1{,}618$$

C_4 = \$1,618 vs. C(4) = \$2,300 acting independently

CITY	USER COST $\$\times10^3$	FRACTION of TOTAL COST	SAVINGS $\$\times10^3$	COST/FLOW RELATIVE TO CITY 4
1	\$1,997	0.29	\$-27	1.54
2	2,134	0.31	166	1.32
3	1,181	0.17	419	1.22
4	1,618	0.23	682	1.00
TOTALS	\$6,930	1.00		

Table 7-5. Cost Allocation for Four Cities

A glance at Table 7-5 reveals quickly that City 1 incurs an additional cost of \$27,000 over what it would have to finance if it acted independently. Thus, City 1 will not join the grand coalition N = {1234}. It is also clear that costs increase upstream from the central treatment facility due to the piping costs. City 4 realizes the largest savings because it does not incur any piping costs, while it reaps the benefits accruing to economies of scale by treatment of all wastes at its facilities. If City 1 drops out leaving 234, then

City 2

$$C_2 = (300)1.0 + \frac{5}{8}(630) + \frac{5}{13}(4{,}600)$$

$$C_2 = 2{,}463$$

$$\text{Savings} = C(2) - C_2 = -163$$

City 2 will not join {234}

City 3

$$C_3 = \frac{3}{8}(630) + \frac{3}{13}(4{,}600)$$

$$C_3 = \$1{,}298$$

$$\text{Savings} = C(3) - C_3 = \$302$$

City 4

$$C_4 = \frac{5}{13}(4{,}600) = \$1{,}769$$

$$\text{Savings} = C(4) - C_4 = \$531$$

The above results seem to indicate that Cities 1 and 2 are likely to join in a coalition to cut costs, while Cities 3 and 4 will likely join to maintain their savings: If {12} + {34} forms, then

City 1

$$C_1 = 290 + \frac{4}{9}(3{,}500) = \$1{,}846$$

$$\text{Savings} = C(1) - C_1 = \$124$$

City 2

$$C_2 = \frac{5}{9}(3{,}500) = \$1{,}944$$

$$\text{Savings} = C(2) - C_2 = \$356$$

City 3

$$C_3 = 403 + \frac{3}{8}(3{,}200) = \$1{,}603$$

$$\text{Savings} = C(3) - C_3 = -3$$

C_3 will not remain in {34}.

City 4

$$C_4 = \frac{5}{8}(3{,}200) = \$2{,}000$$

$$\text{Savings} = C(4) - C_4 = \$300$$

Since City 3 will not join City 4, and the savings incurred by City 1 and City 2 in {12} are large, then the best grouping appears to be

{12} + {3} + {4}

City 1	$C_1 = \$1{,}846$	Savings = \$124
City 2	$C_2 = \$1{,}944$	Savings = \$356
City 3	$C_3 = C(3) = \$1{,}600$	Savings = 0.0
City 4	$C_4 = C(4) = \$2{,}300$	Savings = 0.0

Cities 1 and 2 incur large savings by going together, and Cities 3 and 4 simply act independently. To insure that this is the cost-sharing arrangement most acceptable to the individual cities, we must allocate costs for the rest of the optimal groupings presented in the SUMMARY, part 1(a), page 3.

{123} + {4} :

$$C_1 = 290 + \frac{4}{9}(300) + \frac{4}{12}(4{,}300) = \$1{,}857$$

$$C_2 = \frac{5}{9}(300) + \frac{5}{12}(4{,}300) = \$1{,}958$$

$$C_3 = \frac{3}{12}(4{,}300) = \$1{,}075$$

Costs for Cities 1 and 2 are now higher than before, so they will not likely allow City 3 to join {12}. Also, if total savings are added for grouping [{123} + {4}], they are $25,000 less than total savings incurred by

[{12} + {3} + {4}].

{124} + {3} :

$$C_1 = 290 + \frac{4}{9}(300) + \frac{4}{9}(630) + \frac{4}{14}(4{,}800)$$

$= \$2{,}075$ Savings = $105

$$C_2 = \frac{5}{9}(300) + \frac{5}{9}(630) + \frac{5}{14}(4{,}800)$$

$= \$2{,}231$

Cities 1 and 2 will not allow City 4 to join {12} regardless of its savings.

{134} + {2} :

$$C_1 = 290 + 230 + \frac{4}{7}(525) + \frac{4}{12}(4{,}300) = 2{,}253$$

Savings = -$283

No.

{13} + {24} :

$$C_1 = 290 + 230 + \frac{4}{7}(2{,}950) = \$2{,}206$$

Savings = -$236

No.

{1} + {3} + {24} :

$$C_1 = 300 + 525 + \frac{5}{10}(3{,}800) = \$2{,}725$$

Savings = -$425

No.

{14} + {23} :

$$C_1 = 290 + 230 + 403 + \frac{4}{9}(3{,}500)$$

$C_1 = \$2{,}479$ Savings = -$509

No.

$\{1\} + \{4\} + \{23\}$

$$C_2 = 300 + \frac{5}{8}(3,200) = \$2,300$$

No savings

City 2 would rather join City 1, in which case both incur savings.

SOLUTION

$$\{12\} + \{3\} + \{4\}$$

$$C_1 = \$1,846$$

$$C_2 = 1,944$$

$$C_3 = 1,600$$

$$C_4 = 2,300$$

$$C_1 + C_2 + C_3 + C_4 = \$7,690$$

CITY	USER COST $\$\times10^3$	FRACTION of TOTAL COST	SAVINGS $\$\times10^3$	COST/FLOW RELATIVE TO CITY 2
1	\$1,846	0.24	\$124	1.19
2	1,944	0.25	356	1.00
3	1,600	0.21	0.0	1.37
4	2,300	0.30	0.0	1.18
TOTALS	\$7,690	1.00		

Table 7-6. Revised Cost Allocation for Four Cities

Comparing Table 7-5 (Grand Coalition) with the results shown in Table 7-6, it can be seen that the costs are allocated more evenly. However, the total cost is higher for all four cities. Also, it is obvious that City 1 and City 2 derive all of the savings. Thus, City 3 and City 4 are indifferent to participating in a cooperative venture. Adherence to the use of facilities method rendered the best solution infeasible.

c) Cost Sharing Using Shapley Values

Let $x\{i\}$ = cost allocated to the $i^{\underline{th}}$ waste discharger (City i) if a cooperative arrangement is adopted

$C(N)$ = total cost for the coordinated solution

$$x\{i\} \geqq 0 \text{ for all } i \in N$$

$$\sum_{i \varepsilon N} x\{i\} = C(N)$$

$x\{i\} \leqq C\{i\}$ for all $i \varepsilon N$

N = master set, grand coalition

S = subset

$$\sum_{i \varepsilon S} x\{i\} \leqq C(S) \text{ for all } S \quad N$$

There are $2^n - 1$, or 15 possible coalitions to be analyzed:

{1}, {2}, {3}, {4}

{12}, {13}, {14}, {23}, {24}, {34}

{123}, {124}, {134}, {234}

{1234}

	COALITION	OBJECTIVE FUNCTION	*OPTIMAL SOLUTION, $\$x10^3$	
	C(1)	$f_1(x) = Z_1$	1970	
	C(2)	$f_2(x) = Z_2$	2300	
	C(3)	$f_3(x) = Z_3$	1600	
	C(4)	$f_4(x) = Z_4$	2300	
	C(12)	$f_5(x) = Z_1 + Z_2$	3790	
	C(13)	$f_6(x) = Z_1 + Z_3$	3470	
inessential	C(14)	$f_7(x) = Z_1 + Z_4$	4270	C(1) + C(4)
	C(23)	$f_8(x) = Z_2 + Z_3$	3500	
inessential	C(24)	$f_9(x) = Z_2 + Z_4$	4600	C(2) + C(4)
	C(34)	$f_{10}(x) = Z_3 + Z_4$	3603	
	C(123)	$f_{11}(x) = Z_1 + Z_2 + Z_3$	4890	
	C(124)	$f_{12}(x) = Z_1 + Z_2 + Z_4$	6020	
	C(134)	$f_{13}(x) = Z_1 + Z_3 + Z_4$	5345	
	C(234)	$f_{14}(x) = Z_2 + Z_3 + Z_4$	5530	
	C(1234)	$f_{15}(x) = Z_1 + Z_2 + Z_3 + Z_4$	6930	

Table 7-7. Optimal Solutions to Indicated Minimization Problem.

c(S) = characteristic function for coalition S and represents the minimum cost to that coalition. See values listed in Table 7-7.

$$\phi_i = \underset{S\ N}{\sum} \frac{(S-1)!n-S)!}{n!} \; [C(S) - C(S-\{i\})] \qquad (7\text{-}5)$$

ϕ_i = cost assigned to $i^{\underline{th}}$ city

= Shapley value for N-person games

where $\sum_{i \in N} \phi_i = C(N)$

The core of this game is delimited by:

$$\begin{array}{lllll}
x_1 & & & & \leqq 1970 \\
 & x_2 & & & \leqq 2300 \\
 & & x_3 & & \leqq 1600 \\
 & & & x_4 & \leqq 2300 \\
x_1 & + x_2 & & & \leqq 3790 \\
x_1 & & + x_3 & & \leqq 3470 \\
x_1 & & & + x_4 & \leqq 4270 \\
 & x_2 & + x_3 & & \leqq 3500 \\
 & x_2 & & + x_4 & \leqq 4600 \\
 & & x_3 & + x_4 & \leqq 3603 \\
x_1 & + x_2 & + x_3 & & \leqq 4890 \\
x_1 & + x_2 & & + x_4 & \leqq 6020 \\
x_1 & & + x_3 & + x_4 & \leqq 5345 \\
 & x_2 & + x_3 & + x_4 & \leqq 5530 \\
x_1 & + x_2 & + x_3 & + x_4 & = 6930
\end{array}$$

The Shapley values are:

$$\begin{aligned}
\phi_1 = \frac{1}{4}\,(1970) &+ \frac{1}{12}\,[3790 - 2300] + \frac{1}{12}\,[3470 - 1600] \\
&+ \frac{1}{12}\,[4270 - 2300] + \frac{1}{12}\,[4890 - 3500] \\
&+ \frac{1}{12}\,[6020 - 4600] + \frac{1}{12}\,[5345 - 3603]
\end{aligned}$$

$$+ \frac{1}{4}\,[6930 - 5530] = 1666$$

$$\phi_1 = \$1,666$$

Similarly, $\phi_2 = \$1,898$, $\phi_3 = \$1,220$, and $\phi_4 = \$2,146$

$$\sum_{i=1}^{4} \phi_i = \$6,930$$

Table 7-8 shows a fairly even distribution of costs and savings among the grand coalition participants.

CITY	USER COST, ϕ_i $\$\times 10^3$	FRACTION of TOTAL COST	SAVINGS
1	\$1,666	0.24	\$304
2	1,898	0.27	402
3	1,220	0.18	381
4	2,146	0.31	154
TOTAL	6,930		

Table 7-8. Cost Allocation Using Shapley Value

The Shapley value solution falls in the core:

$$\begin{aligned}
x_1 &= 1666 < 1970\\
x_2 &= 1898 < 2300\\
x_3 &= 1220 < 1600\\
x_4 &= 2146 < 2300
\end{aligned}$$

$$\begin{aligned}
x_1 + x_2 &= 3564 < 3790\\
x_1 + x_3 &= 2886 < 3470\\
x_1 + x_4 &= 3812 < 4270\\
x_2 + x_3 &= 3118 < 3500\\
x_2 + x_4 &= 4044 < 4600\\
x_3 + x_4 &= 3366 < 3603
\end{aligned}$$

$$\begin{aligned}
x_1 + x_2 + x_3 &= 4784 < 4890\\
x_1 + x_2 + x_4 &= 5710 < 6020\\
x_1 + x_3 + x_4 &= 5032 < 5345\\
x_2 + x_3 + x_4 &= 5264 < 5530\\
x_1 + x_2 + x_3 + x_4 &= 6930
\end{aligned}$$

JOINT PRODUCTS AND BY-PRODUCTS OF A STP

In a manufacturing operation, if two or more products are produced from the same unit process, a cost allocation problem arises. There are specific costs identifiable to one product and joint costs not identifiable to a single product. How does a manufacturing firm decide what proportion of the joint manufacturing costs a particular product will pay?

One method has been to allocate all the joint costs to one product. This product is called the main product of the process. The other product(s) manufactured are termed by-products. They are only charged their specific costs. This method will be called by-product costing.

Another method (joint product costing), is to treat all the products as joint products of the process. Joint products have roughly the same value. By-products do not have values of the same magnitude as joint products.

The joint costs of the joint products are allocated in one of two ways. A physical measure, such as pounds of product produced, may be used to allocate joint costs. Or, a monetary value may be calculated for the products and joint costs allocated according to the monetary return of the product. We will use a physical measure, pounds per day removed, to allocate the costs of a sewage treatment plant (STP).

The sewage treatment process can be considered a manufacturing operation. A raw material, wastewater, is transformed by physical, chemical and biological processes into a finished product or products.

The product or products of a STP will be considered to be water satisfying the State effluent standards. If the State requires an effluent with a BOD_5 less than 5 mg/l then there is only one relevant product of the STP, treated water with a BOD_5 less than 5 mg/l. If the State requires the STP's effluent to have an SS concentration less than 5 mg/l, then the STP has two products.

POLLUTANT PRICING IN A SEWAGE TREATMENT PLANT

An engineer desiring to estimate the cost of removing a pollutant in a sewage treatment plant (STP) would be hard pressed to find an acceptable method. Most units in a STP are multipurpose, i.e., they are designed to remove more than one type of pollutant. How does the engineer allocate the costs of treatment among the many purposes? For example, what percentage of the costs of a primary settling basin are chargeable to BOD_5 removal when both BOD_5 and suspended solids are removed? Or consider an activated sludge unit that removed soluble BOD_5; has luxury uptake of phosphorus; and removes suspended solids in the secondary clarifier? Should BOD_5 removal be charged with the whole cost of the activated sludge unit? If not, then by what measure should the engineer allocate the costs among the various purposes?

Game theoretic concepts can be used to estimate pollutant removal costs. Specifically, the Shapley value, and a new game theoretic concept, the simple Shapley value, will be studied as solutions to the pollutant removal cost game.

The players in this game are pollutants, e.g., BOD_5, suspended solids, total phosphorus, and nitrogen. Each pollutant must be removed to the extent that its effluent concentration falls below a prespecified level. The characteristic function cost of a coalition will be the least cost sewage treatment system that satisfies the effluent concentration limits for all members of the coalition.

An example four player game will be considered. It will first be solved using the Shapley value. Then utilizing the special structure of the characteristic function costs, the simplified shapley value will be calculated. It will be found that the Shapley value assigned cost and the simplified Shapley value assigned cost for each pollutant are the same. The advantages of this result will then be discussed.

Description of pollutant removal cost game

The four groups of the pollutant removal cost game are:

Group 1	BOD_5
Group 2	suspended solids
Group 3	total phosphorus
Group 4	nitrogen

The influent concentrations of the four pollutants are assumed to be the same as those in Table 7-9. The concentrations are:

Pollutant	Influent Concentration
BOD_5	210 mg/1
suspended solids	230 mg/1
total phosphorus	11 mg/1
nitrogen	20 mg/1

The sewage treatment plant is assumed to have a design and average flow of 20 MGD. The cost functions and removal rates for all systems is the same as in Table 7-9.

The maximum effluent concentrations allowed for the pollutants are:

Pollutant	Effluent Concentration
BOD_5	10 mg/1
suspended solids	5 mg/1
total phosphorus	0.5 mg/1
nitrogen	2 mg/1

There are 15 possible coalitions among four groups. The characteristic function cost of a coalition is the least costly system that satisfies all the

Table 7-9 Wastewater Treatment System Production and Cost Function[1]
Values of a in $y_j = a_{ij}\ x_i\ (x_i = 0,1)$

System Number i =	1 BOD_5	2 TSS	3 COD	4 Total P	5 NH_3-N	6 UOD[2]	Description of System	Annual Cost[3], $C = aQ^b$ a	b
-	1.00	1.00	1.00	1.00	1.00	1.00	Raw Waste Characteristics	-	-
1	.38	.57	.38	.18	0	.30	Primary	.17	.62
2	.52	.78	.54	.82	0	.41	Primary and Metal Salt Addition ($FeCl_3$)	.23	.71
3	.79	.74	.78	.27	.10	.70	Primary and Trickling Filter	.24	.67
4	.88	.87	.88	.82	.10	.63	Primary and Trickling Filter with Metal Salt ($FeCl_3$)	.31	.75
5	.90	.91	.89	.36	.15	.74	Primary and Activated Sludge	.26	.69
6	.93	.93	.91	.82	.15	.75	Primary, Activated Sludge, Metal Salt (Alum)	.30	.72
7	.95	.91	.91	.27	.90	.94	Primary, Activated Sludge/Nitrification	.28	.67
8	.95	.91	.89	.27	.95	.95	Primary, Activated Sludge, Nitrification Denitrification	.40	.71
9	.95	.96	.89	.91	.90	.94	Primary Metal Salt Addition (Alum), Activated Sludge/Nitrification, Filtration	.36	.71
10	.98	.98	.94	.91	0	.76	Preliminary, Two-Stage Lime, Filtration, Carbon Adsorption	.80	.74
11	.98	.98	.92	.95	.95	.97	Primary Metal Salt Addition (Alum), Activated Sludge, Nitrification, Denitrification, Polymer, Filtration	.54	.72
12	.99	.98	.98	.95	.95	.97	Primary Metal Salt Addition (Alum), Activated Sludge, Nitrification, Denitrification, Polymer, Filtration, Carbon Adsorption	.70	.71

Note: Treatment systems include disinfection, sludge handling, miscellaneous structures, and support personnel.

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[1] Source: EPA, Areawide Assessment Procedures Manual. App. H., EPA-600/9-76-014, Cincinnati, OH, 1976.

[2] UOD = Ultimate Oxygen Demand = ($1.5 \times BOD_5$) + ($4.57 \times NH_3$-N).

[3] Treatment costs are annual costs in millions of 1976 dollars.

effluent standards of the members of the coalition. As an example of the calculation of the characteristic function cost, consider group 1's cost of separate action. Group 1, desiring an effluent concentration below 10 mg/1, seeks the system of least cost among the subset of the 12 systems that have an effluent BOD_5 below 10 mg/1. Group 1 therefore chooses system 7 from the subset of systems 7, 8, 9, 10, 11, and 12 since it is the least cost system.

Similarly, the characteristic function cost of the other 14 coalitions can be calculated. They are presented in Table 7-10.

Coalition	Feasible Systems	Optimal System	Cost($/year)
C(1)	7,8,9,10,11,12	7	2,080,000
C(2)	10,11,12	11	4,670,000
C(3)	11,12	11	4,670,000
C(4)	8,11,12	8	3,355,000
C(12)	10,11,12	11	4,670,000
C(13)	11,12	11	4,670,000
C(14)	8,11,12	8	3,355,000
C(23)	11,12	11	4,670,000
C(24)	11,12	11	4,670,000
C(34)	11,12	11	4,670,000
C(123)	11,12	11	4,670,000
C(124)	11,12	11	4,670,000
C(134)	11,12	11	4,670,000
C(234)	11,12	11	4,670,000
C(1234)	11,12	11	4,670,000

Table 7-10. Characteristic Function Costs

Shapley Value

The Shapley value of the four group pollutant removal cost game is calculated below using equation 7-5.

$$\begin{aligned}\phi(1) = {} & 1/4c(1) + 1/12\,[c(12) - c(2)] + 1/12[c(13) - c(3)] \\ & + 1/12[c(14) - c(4)] + 1/12[c(123) - c(23)] \\ & + 1/12[c(124) - c(24)] + 1/12[c(134)-c(34)] \\ & + 1/4[c(1234) - c(234)]\end{aligned}$$

$$\begin{aligned}\phi(1) = {} & 1/4(2{,}080{,}000) + 1/12(0) + 1/12(0) \\ & + 1/12(0) + 1/12(0) + 1/12(0) + 1/12(0) \\ & + 1/4(0)\end{aligned}$$

$$\phi(1) = \$520{,}000/\text{year}$$

$$\begin{aligned}\phi(2) = {} & 1/4c(2) + 1/12\,[c(12) - c(1)] + 1/12[c(23) - c(3)] \\ & + 1/12[c(24) - c(4)] + 1/12[c(123) - c(13)] \\ & + 1/12[c(124) - c(14)] + 1/12[c(234)-c(34)] \\ & + 1/4[c(1234) - c(134)]\end{aligned}$$

$$\begin{aligned}\phi(2) = {} & 1/4(4{,}670{,}000) + 1/12(4{,}670{,}000 - 2{,}080{,}000) \\ & + 1/12(0) + 1/12(4{,}670{,}000 - 3{,}355{,}000) + 1/12(0) \\ & + 1/12(4{,}670{,}00 - 3{,}355{,}000) + 1/12(0) \\ & + 1/4(0)\end{aligned}$$

$\phi(2) = \$1{,}602{,}500$/year.

$$\begin{aligned}\phi(3) = {} & 1/4c(3) + 1/12\,[c(13) - c(1)] \\ & + 1/12[c(23) - c(2)] + 1/12[c(34) - c(4)] \\ & + 1/12[c(123) - c(12)] + 1/12[c(134) - c(14)] \\ & + 1/12[c(234)-c(24)] \\ & + 1/4[c(1234) - c(124)]\end{aligned}$$

$$\begin{aligned}\phi(3) = {} & 1/4(4{,}670{,}000) + 1/12(4{,}670{,}000 - 2{,}080{,}000) \\ & + 1/12(0) + 1/12(4{,}670{,}000 - 3{,}355{,}000) \\ & + 1/12(0) + 1/12(4{,}670{,}00 - 3{,}355{,}000) \\ & + 1/12(0) + 1/4(0)\end{aligned}$$

$\phi(3) = \$1{,}602{,}500$/year.

$$\begin{aligned}\phi(4) = {} & 1/4c(4) + 1/12\,[c(14) - c(1)] \\ & + 1/12[c(24) - c(2)] + 1/12[c(34) - c(3)] \\ & + 1/12[c(124) - c(12)] + 1/12[c(134) - c(13)] \\ & + 1/12[c(234)-c(23)] \\ & + 1/4[c(1234) - c(122)]\end{aligned}$$

$$\begin{aligned}\phi(4) = {} & 1/4(3{,}355{,}000) + 1/12(3{,}355{,}000 - 2{,}080{,}000) \\ & + 1/12(0) + 1/12(0) + 1/12(0) \\ & + 1/12(0) + 1/12(0) + 1/12(0)\end{aligned}$$

$\phi(4) = \$945{,}000$/year

The ϕ_i's sum to $4,670,000 which is the grand coalition cost.

As shown above the calculation for even a four purpose game is tiresome. For games with five or more purposes the calculations become increasingly difficult. Fortunately, a special quality of the characteristic function costs allows an assumption that leads to a simple expression for the Shapley value. The special quality is that the purpose with the largest cost of separate action in a coalition determines the characteristic function cost of the whole coalition. For example, in coalition 124 purpose 2 determines the characteristic function cost of the coalition since he has the largest cost of separate action. The cost of coalition c(124) is therefore the same as c(2), or $4,670,000 per year.

The 15 coalitions have only 3 distinct characteristic function costs. Associated with the 3 distinct costs are 3 types of purposes. The costs may be ordered from smallest to largest. After ordering the costs in this game we have: c_1, or \$2,080,000, associated with group 1, c_2, or \$3,355,000, associated with group 4, and c_3, or \$4,670,000, associated with groups 2 and 3.

Littlechild and Owen (5) prove that the Shapley value, equation 7-5, may be represented in this special case by the following equation:

$$\phi_i = \phi_{i-1} + (c_i - c_{i-1})/r_i \qquad (7\text{-}6)$$

where c_i = cost associated with group type i,

c_{i-1} = cost associated with group type (i-1),

ϕ_i = Shapley value assigned cost to group type i, and

ϕ_i = Shapley value assigned cost to group type (i-1).

r_i is explained below.

In general there are m group types. Each group type has n_i players associated with it. In this pollutant removal cost game group types one and two have only one member associated with them while group type three has two members associated with it. r_i can now be defined to be

$$r_i = \sum_{i=k}^{m} n_i \text{ for } k = 1, 2, \ldots m$$

For this game

$r_i = 1 + 1 + 2 = 4$

$r_2 = 1 + 2 = 3$, and

$r_3 = 2$.

The simple Shapley value assigned costs can now be calculated.

$$\phi(1) = c(1)/r_1$$

$$\phi(1) = 2{,}080{,}000/4 = \$520{,}000$$

$$\phi(2) = \phi(1) + \frac{c(2) - c(1)}{r_2}$$

$$\phi(4) = 520{,}000 + \frac{3{,}355{,}000 - 2{,}080{,}000}{3}$$

$$\phi(4) = \$945{,}000$$

$$\phi(3) = \phi(2) + \frac{c(3) - c(2)}{r_3}$$

$$\phi(3) = 945{,}000 + \frac{4{,}670{,}000 - 3{,}355{,}000}{2}$$

$$\phi(3) = \$1{,}602{,}500 = \phi(2)$$

The assigned costs to the four groups are shown in Table 7-11. Costs are all \$/year.

Group	Shapley Value	Simple Shapley Value
1	520,000	520,000
2	1,602,500	1,602,500
3	1,602,500	1,602,500
4	945,000	945,000

Table 7-11. Assigned Costs Using the Shapley Value and the Simplified Shapley Value.

As shown in Table 7-11, the Shapley value assigned costs and the simple Shapley value assigned costs are the same. The amount of effort in their respective calculation are vastly different, however. The Shapley value involves 2^n-1 coalitions while the simple Shapley value will have less than n group types. For large games this results in a significant savings of effort.

Consider what would happen if we added two more groups to the pollutant removal cost game. We must now find the characteristic function cost of 63 coalitions before even attempting the complicated mechanics of calculating the Shapley value assigned cost. By contrast the computing of the simple Shapley value would take no more than two extra steps.

Clearly the computational advantage lies with the simple Shapley value in games of this type. But how common are situations that can be modeled as games with the necessary characteristic function cost structure?

Basically any situation that has

1) a multipurpose facility;

2) the players cost of separate action ordered from least to most costly; and

3) the facility built for any player also satisfies the requirements of the players with cheaper costs;

may be modeled as a game that could be solved using the simple Shapley value.

The above was true in the pollutant removal cost game since the system built to satisfy the suspended solids and total phosphorus standards also met the requirements of BOD_5 and nitrogen. Additionally, the system for nitrogen satisfied the BOD_5 standards.

Another example of this type of game situation is described by Littlechild and Thompson (6). The problem was to determine airport landing charges at Birmingham airport in England. The game consisted of 11 types of aircraft and 13,572 landings or players.

Since the cost of providing an airport runway essentially depends upon the largest type of aircraft to land there, they suggested the following scheme.

> "Divide the cost of catering for the smallest type of aircraft equally among the number of landings of all aircraft."
>
> "Divide the incremental cost of catering for the second smallest type of aircraft (above the cost of the smallest type) equally among the number of landings of all but the smallest type of aircraft. Continue in this manner until finally the incremental cost of the largest type of aircraft (above the cost of the second largest type) is divided equally among the number of landings made by the largest aircraft type."

This of course is the simple Shapley value assigned cost for the airport landing fee game. It also points out the utility of the simple Shapley value in large, (13,572 players) games.

Returning to the pollutant removal cost game we can consider the possible benefits of using the simple Shapley value.

It eliminates the need for a physical measure of use, such as pounds removed per day, that a cost accountant would require to estimate pollutant charges.

It provides a means of estimating the true cost of a policy. For example, in estimating the cost of a policy that requires less than 7 mg/l total phosphorus in the STP's effluent it may now be found by considering only the incremental cost. The Shapley value cost would be much higher and may in fact represent the true costs of removal.

The main usage of the simple Shapley value lies in easing the rigors of calculating a game theoretic solution to large games. It is compatible with the MCRS method since the solutions are identical for convex cost games.

WATER RESOURCES DEVELOPMENT IN JAPAN

Description of the Problem

Suzuki and Nakayama (7) used the nucleolus, a game theoretic solution concept, to assign costs in a five group water resource development game along Japan's Sakawa and Sagami Rivers. The five groups are the:

River Sakawa agricultural association (1),
River Sagami agricultural association (2),
Kanagawa city water service (3),
Yokahama city water service (4), and
Kawasaki city water service (5).

The agricultural associations have an excess quantity of water that could be made available to the city water services. The city water services have an expected future additional water need that could be met either by diverting water from the agricultural associations or by constructing dams on the Sakawa and Sagami Rivers.

The costs associated with municipal water development include (1) constructing dams to meet part of the city water service's future needs; and/or (2) diverting water from the agricultural associations; and/or (3) compensating the agricultural associations for any decrease in production resulting from the water diversions.

The characteristic function, C(S), is defined as the cost of coalition S. The costs (all x 10^8 yen) for all possible coalitions of the five groups are shown below.

A. Single group coalition

C(1) = C(2) = 0
C(3) = 489.7
C(4) = 747.6
C(5) = 749.8

B. Two group coalitions

C(12) 0
C(13) = 440
C(14) = 700.5
C(15) = 694
C(23) = 486.4
C(24) = 546.3
C(25) = 512.2
C(34) = 1106.5
C(35) = 1108.3
C(45) = 1209

C. Three group coalitions

C(123) = 440
C(124) = 518.1
C(125) = 490.2
C(134) = 998.0
C(135) = 961.5
C(145) = 1069.2
C(234) = 948.2
C(235) = 950.7
C(245) = 1069.2
C(345) = 1554.3

D. **Four group coalitions**

C(1234) = 865.4	C(1345) = 1424.6
C(1235) = 803.8	C(2345) = 1424.3
C(1245) = 940.9	

E. **Five group coalition**

C(12345) = 1307.9

Solution Concepts

The nucleolus (8) maximizes the minimum happiness of any coalition. The measure of the happiness of coalition S, called e_s, is the savings realized by S from cooperation with the other (N-S) members of the grand coalition.

Let e_1 denote the minimum savings of any S. Then,

$$e_1 = C(S) - \sum_{i \in S} x_i \qquad (7\text{-}7)$$

where e_1 = minimum savings to any coalition,

C(S) = characteristic function cost, and

$\sum_{i \in S} x_i$ = sum of the allocated costs of all the members of S.

The nucleolus is found by solving the following linear programming problems. First,

$$\text{maximize } e_1 \qquad (7\text{-}8)$$

Subject to: $e_1 + x_i \leqq C(i)$ $\quad \forall\ i = 1,2,\ldots n.$

$e_1 + \sum_{i \in S} x_i \leqq C(S)$ $\quad \forall\ S\ \ N$

$\sum_{i \in N} x_i = C(N)$

where C(N) = characteristic function cost of the grand coalition.

The nucleolus is calculated by sequentially solving for e_1 then e_2, e_3, etc., where e_i is the i^{th} smallest savings to any coalition. The nucleolus is found after at most n-1 linear programs. It satisfies all the core conditions, always exists, and is unique.

In this case, the nucleolus was found after solving just one linear program. Indeed, Suzuki and Nakayama present the following single equation solution for this game.

$$x^*(i) = [C(N) - C(N-\{i\})] - n^{-1} \sum_{i \in N} (C(N) - C(N-\{i\})) - C(N) \qquad (7\text{-}9)$$

where $x^*(i)$ = cost assigned to player i,

$C(N)$ = characteristic function cost of the grand coaliton, and

$C(N-\{i\})$ = characteristic function cost for the n-1 players with i the player excluded.

Equation (7-9) is identical to an accepted cost allocation formula in water resources planning (9).

Note that

$$C(N) - C(N-\{i\}) = SC(i) \qquad (7\text{-}10)$$

where $SC(i)$ = separable cost of group i, and

$$\sum_{i \in N} (C(N) - C(N-\{i\})) - C(N) = -NSC \qquad (7\text{-}11)$$

where NSC = nonseparable costs. Thus, equation (7-9) is identical to the cost allocation formula

$$x^*(i) = SC(i) + 1/n(NSC) \qquad (7\text{-}12)$$

wherein each group pays his separable cost plus an equal share of the nonseparable cost.

The reason the nucleolus is given by the equal share formula is due to the fact that all the $C(N-\{i\})$ coalitions have savings of e_1, the minimum savings of any coalition. Generally, the $C(N-\{i\})$ coalitions will not have the same savings and equation (7-12) will not yield the nucleolus.

Results

The assigned costs as determined by the nucleolus, and the MCRS and SCRB methods, are shown in Table 7-12.

Method	1	2	3	4	5
Nucleolus	-70.9	-71.2	412.5	549.5	488.0
MCRS	-78.6	-78.8	406.8	555.5	503.0
SCRB	-87.2	-87.4	397.8	565.1	519.6

Table 7-12. Cost Assigned to Group i(10^8 yen)

The upper and lower bounds on the costs assigned to the five groups from the MCRS programs are shown below.

$$\begin{aligned} -116.4 &\leq x(1) \leq 0 \\ -116.7 &\leq x(2) \leq 0 \\ 367.0 &\leq x(3) \leq 489.7 \\ 504.1 &\leq x(4) \leq 663.0 \\ 442.5 &\leq x(5) \leq 628.9 \end{aligned}$$

The maximum assignable costs to groups 4 and 5 are less than their costs of independent action. The maximum assignable costs to groups 1, 2 and 3 are their costs of independent action.

Since the minimum costs of the MCRS method are identical to the separable costs of the SCRB method, the only difference between the two methods is in the upper bounds.

To see why the MCRS upper bounds are more reasonable than the SCRB upper bounds, consider the following argument. Let groups 1 thru 4 pay only their separable costs with group 5 paying the balance. The total cost is 1307.9×10^8 yen. This is much less than group 5's go it alone cost of 749.8×10^8 yen.

The reason group 5's upper bound, 628.9×10^8 yen, is even lower than 669.9×10^8 yen is due to the geometry of the core. When the program tries to maximize x(5) the constraints on the coalitions C(1235), C(1245), and C(1345) prohibit group 5 from being charged more than 628.9×10^8 yen. If group 5 were charged more, group rationality would be violated since the three coalitions would pay more than they would by independent action.

The costs assigned by the three methods are similar. The reason is due to the small amount of non-separable costs. The NSC is 227.4×10^8 yen, about 17% of the grand coalition cost. Each method assigns at least his separable cost to every group.

Thus, even though the three methods differ widely in the percentage each group pays of the NSC, the end result is similar cost allocations due to the small NSC.

The percentage each group pays of the NSC under the three methods is given in Table 7-13.

	Method		
Group	Nucleolus	MCRS	SCRB
1	20%	16.6%	12.85%
2	20%	16.6%	12.85%
3	20%	17.5%	13.5%
4	20%	22.7%	26.9%
5	20%	26.9%	33.9%
Total	100	100.0	100.0

Table 7-13. Percentage of Cost Paid by Each Group

The difference between the SCRB and MCRS costs is solely attributable to the difference in upper bounds.

EIGHT MEMBER REGIONAL WASTEWATER TREATMENT SYSTEM

This section presents a case study of a regional wastewater management problem analyzed by Orlando et al. (10). The study area is the Lower Meramec River near St. Louis, Missouri. All of the waste sources are combined into the eight equivalent point sources shown in Table 7-14.

Orlando et al. present the optimal solution for the forty-six coalitions which were found to be economically attractive. Our analysis of these data indicated that twelve of these coalitions were inessential, i.e., $C(S) + C(T) \leq C(S \cup T)$. The cost for the eight polluters and the remaining essential coalitions are presented in Table 7-15. The optimal solution is the least costly combination of treatment plants(s) and piping for the coalition. The required level of pollution control is assumed to be the required level for the downstream member of the coalition. A receiving water model was run to verify that water quality standards would be met. Treatment facilities were located only at cities so that the industrial users could share in the public subsidy of the treatment works. The Federal Government and the State of Missouri pay 75% and 15% of the cost of construction, respectively. Thus, the subsidized cost for each polluter is 10% of the construction costs, and all of the operation and maintenance costs.

Influence of Differential Subsidies

The first problem to be examined is how subsidies might influence the analysis. In the example, the industrial waste dischargers receive no subsidy

Number	Source	River Mile	Facility Capacity, Million Gal./Day	Required % Pollution Control BOD	Required % Pollution Control Ammonia
1	Fishpot Creek	22.40	5.0	96	85
2	Absorbent/V.P.	21.52	3.4	96	85
3	Grand Glaize Creek	20.38	10.3	98.5	85
4	Chrysler Corporation	20.36	2.2	BPT[a]	none
5	Fenton	15.20	3.2	96	85
6	Saline Creek	10.42	2.7	96	85
7	Mattese Creek	4.76	5.6	96	85
8	Arnold	2.00	2.6	91	none

[a] BPT - Best Practicable Treatment

Table 7-14. List of Waste Sources, Meramec Regional System (9)

		Cost: $\$ \times 10^3$/yr				
Number	Coalition	Amortized Capital	O & M	Total	Subsidized Total[a]	Savings $\$ \times 10^3$/yr
1	1	547	576	1123	631	0
2	2	402	387	789	427	0
3	3	1270	1320	2590	1447	0
4	4	375	165	540	202	0
5	5	530	360	890	413	0
6	6	447	320	767	365	0
7	7	594	546	1140	605	0
8	8	382	188	570	226	0
9	34	1540	1510	3050	1664	80
10	35	1710	1010	2720	1181	760
11	78	840	430	1270	514	440
12	135	2120	1260	3380	1472	1223
13	235	2000	1310	3310	1510	959
14	345	1860	1120	2980	1306	1040
15	1234	2280	2240	4520	2468	522
16	1235	2360	1420	3780	1656	1612
17	1345	2250	1360	3610	1585	1533
18	2345	2120	1290	3410	1502	1399
19	3456	2360	1230	3590	1466	1197
20	3578	3080	900	3980	1208	1210
21	5678	1980	650	2630	848	737

[a] Subsidized total costs = 10% of amortized capital costs plus operation and maintenance costs.

Table 7-15. Costs and Savings for 41 Coalitions (10)

		Cost: $\$ \times 10^3$/yr.				
Number	Coalition	Amortized Capital	O & M	Total	Subsidized Total	Savings $\$ \times 10^3$/yr
22	12345	2360	1530	3890	1766	2042
23	13578	3480	1100	4580	1448	1733
24	23578	3440	1000	4440	1344	1539
25	35678	3480	960	4440	1308	1517
26	123578	3950	1170	5120	1565	1982
27	134567	3670	1820	5490	2187	1560
28	134578	3690	1100	4790	1469	2063
29	135678	3870	1120	4990	1507	2090
30	145678	3350	900	4250	1235	780
31	234567	3510	1740	5250	2091	1466
32	234578	3630	1110	4740	1473	1779
33	235678	3784	1097	4881	1475	1865
34	345678	3820	1080	4900	1462	1597
35	1234567	3960	1220	5180	1616	2659
36	1234578	4076	1242	5318	1650	2324
37	1235678	3890	1240	5130	1629	2739
38	1245678	3990	980	4970	1379	849
39	1345678	3970	1190	5160	1587	2460
40	2345678	3850	1160	5010	1545	2276
41	12345678	4120	1300	5420	1712	2989

[a] Subsidized total costs = 10% of amortized capital costs plus operation and maintenance costs.

Table 7-15. Costs and Savings for 41 Coalitions (concluded) (10)

if they treat their waste. However, if they join with a city, then the entire system receives the 90% subsidy of capital costs. This is an improper way for a subsidy program to work since it encourages coalitions which are economically inefficient but financially attractive because of the subsidy. Related problems result when the state and/or EPA refuse to subsidize individual cities but will subsidize "regional" systems. The individual cities understandably seek to minimize their net (with subsidy) costs whereas the coordinating group seeks to minimize total (unsubsidized) costs. Thus, subsidies should not discriminate against certain coalitions, e.g., industries going alone in this case.

The next problem deals with the fact that only capital costs are subsidized. Figure 7-4 shows a hypothetical isoquant for 90% pollution control as a function of operation and maintenance costs and amortized capital costs. The designer chooses the solution which minimizes total costs.

Without subsidies the problem is to

$$\text{minimize } Z = C_{om} + C_c$$

$$\text{subject to } y = f(C_{om} + C_c)$$

This problem can be solved graphically by picking an arbitrary value of Z, say Z = \$50 in this example, and plotting the objective function, i.e., $C_{om} = \$50 - C_c$. This is a straight line which intercepts the C_{om} axis at \$50 and has a slope of -1.0. Since the trial value of Z was arbitrary, the optimal solution can be found by finding the point of tangency between a slope of -1.0 and the isoquant as shown in Figure 7-4. The resulting least cost solution is Z = \$57.00.

However, if as in the Meramec River Basin example, 90% of the amortized capital costs are paid by the state and federal government, then the optimization problem facing the cities is to

$$\text{minimize } Z = C_{om} + 0.1\ C_c$$

$$\text{subject to } y = f(C_{om},\ C_c)$$

This problem can be solved graphically as above by picking an arbitrary Z, say \$50, and plotting $C_{om} = \$50 - 0.1\ C_c$. Now the optimal solution is the point of tangency of a slope of -10.0 with the isoquant. This shifts the optimal solution dramatically because it is now in the city's best interest to favor the capital intensive solution shown in Figure 7-4. The total unsubsidized cost for this solution is \$90. However, because of the subsidy, the city only pays \$14.40. However, this solution is wasteful from society's viewpoint since total costs increase by \$33.00 (\$90.00 vs. \$57.00).

The subsidy distorts the economic optimum because it encourages substitution of capital costs for operation and maintenance costs. No distortion would occur if the subsidy is applied to total costs because the slope of the tangency line is unaffected, i.e., the slope remains -1.0. Thus, in principle, the subsidizing agencies should subsidize total costs only and not

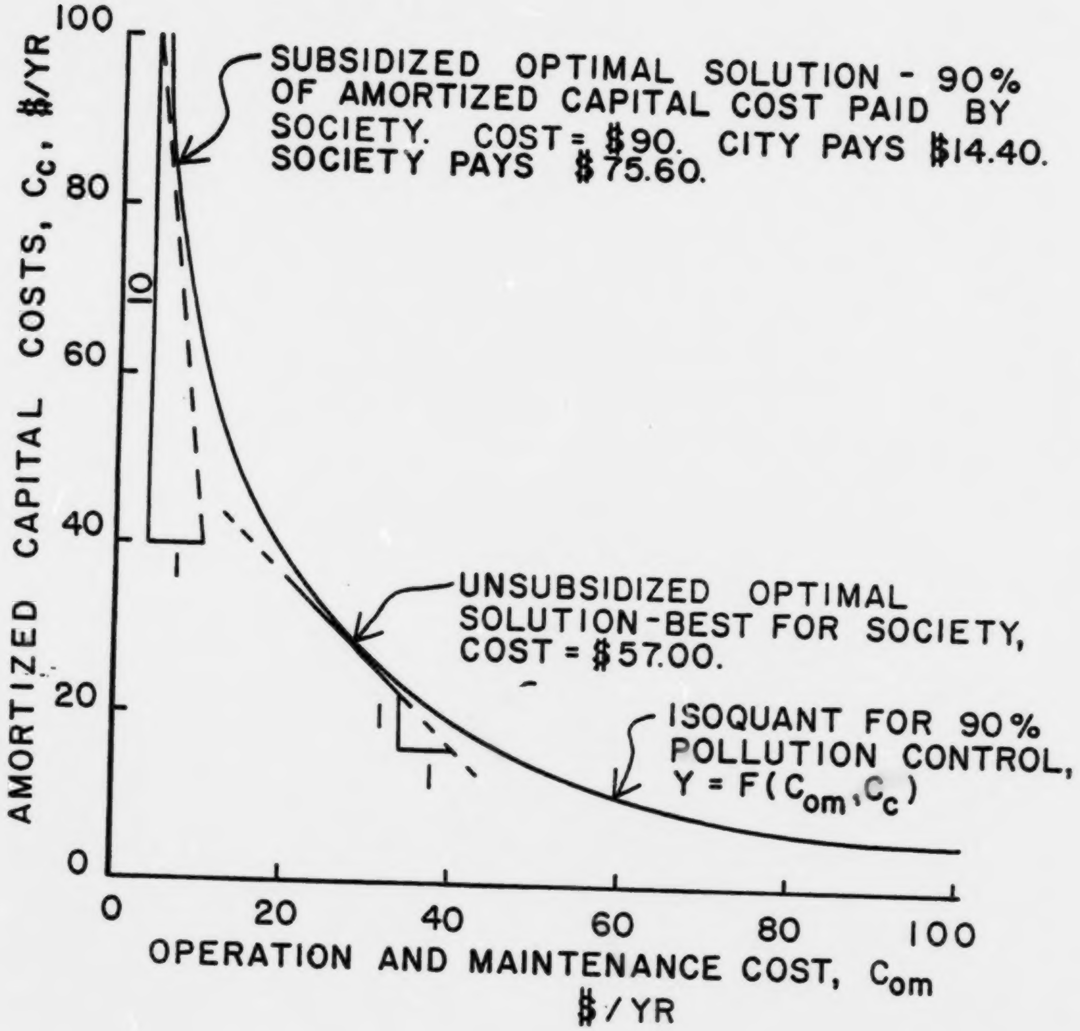

Figure 7-4. Effect of Subsidy on Selected Optimal Solution.

subsidize only selected alternatives. Similar situations arise in many areas of public investment, e.g., structural vs. non-structural flood control measures. The seemingly obvious answer is to permit uniform overall subsidies. However, this is very difficult to implement in public decision making since the subsidizing agencies can only guarantee money for the current fiscal year. They are not empowered to commit the government to a long-term obligation for operation and maintenance costs. They can and do make one time allotments for construction of facilities. Thus, all we can do is to require that the applicants submit optimal unsubsidized solutions and not try to exploit the fact that differential subsidies are available. The interested reader should refer to Marshall and Ruegg (12) for an in-depth treatment of this subject.

Unsubsidized or Subsidized Costs?

An essential ingredient of the cost sharing analysis is a definition of the characteristic function which unambiguously states how the cost for any coalition is to be determined. In this example, if the coalition contains a city, it is entitled to a 90% subsidy of the costs of the capital costs. Indeed, some of these coalitions are essential (save money) only because of the subsidy. Here again, a possible distortion of costs arises. Furthermore, it is not a simple linear transformation of unsubsidized costs so its ramifications are not obvious. To illustrate the differences in assigned costs, the cost sharing was done using subsidized and unsubsidized costs. The results are shown in Table 7-16. The impact of using subsidized vs. unsubsidized costs is significant. Although the total savings are similar, the distribution varies widely. Since the subsidies can distort the economic calculations and the cost sharing it seems prudent to use unsubsidized costs in defining the characteristic function. Then, we are assured that the value of the characteristic function is economically efficient.

	Savings Using Unsubsidized Costs: $\$ \times 10^3$/yr.		Savings Using Subsidized Costs: $\$ \times 10^3$/yr.	
Group	MCRS	Shapley Value	MCRS	Shapley Value
1	331	280	212	220
2	246	204	317	470
3	996	840	570	536
4	117	102	214	412
5	412	796	995	745
6	235	199	123	150
7	499	439	764	393
8	154	130	29	300
TOTALS	2990	2990	3226	3226

Table 7-16. Annual Savings Using Different Cost Sharing Formulas and Subsidized vs. Unsubsidized Costs.

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APPENDIX

Computing the Nucleolus and MCRS Using the MPS Linear Programming Code

In Chapter 6 the nucleolus was computed both graphically and by a desktop iterative method. This is acceptable and even time saving method for small two and three person games. But if we were required to compute the nucleolus of games with more than four players the calculations would quickly become time consuming and subject to error. If the nucleolus of a large game is desired it should be solved as a series of linear programs.

We will describe the solution of a three person game as an illustration of the linear programming method. The nucleolus will be solved for first. The MCRS solution will then be found and the solutions compared for ease of programming and CPU (Central Processing Unit) time.

The linear programming code used is IBM's Mathematical Programming System-Extended (MPSX), see (1). The University of Florida has an Amdahl 470 V/6 II central processing unit.

The example cost allocation game involves three states of India that are cooperating in a regional electric power generating scheme involving hydroelectric, nuclear, and oil burning power plants (2). The states are Andhra Pradesh, Tamil Nadu, and Kerala-Mysore. They will be abbreviated as AP, TN, and KM respectively.

The characteristic function costs are as follows: (all x 10^6 rupees)

C(AP) = 1870 C(AP,TN,KM) = 6530
C(TN) = 5330
C(KM) = 860
C(AP, TN) = 6990
C(AP, KM) = 1960
C(TN, KM) = 5020

This game has a convex core. However, it is inconvenient to work with characteristic function costs since KM's fair cost allocation will be negative. This results from the fact that C(AP,TN) is greater than the grand coalition cost. AP and TN pay KM to join the grand coalition. This is the result of the large hydroelectric potential of KM. The hydroelectric plants are significantly cheaper than the other power generating plants.

Therefore, we will transform the cost game into a value game by subtracting the characteristic function cost of a coalition from the sum of the coalition members go it alone cost. This is illustrated below:

v(AP) = 1870 - 1870 = 0
v(TN) = 5330 - 5330 = 0
v(KM) = 860 - 860 = 0
v(AP,TN) = 5330 + 1870 - 6990 = 210

$v(AP,KM) = 1870 + 860 - 1960 = 770$
$v(TN,KM) = 5330 + 860 - 5020 = 1170$
$v(AP,TN,KM) = 5330 + 860 + 1870 - 6530 = 1530$

where $v(S)$ is the characteristic function value of coalition S.

The linear programming formulation of the nucleolus for a value game is:

$$\text{Maximize } e_1$$

subject to:

$$\begin{aligned}
&AP - e_1 \geq 0\\
&TN - e_1 \geq 0\\
&AP + TN - e_1 \geq 210\\
&AP + KM - e_1 \geq 770\\
&TN + KM - e_1 \geq 1170\\
&AP + KM + TN = 1530\\
&AP,\ TN,\ KM \geqq 0
\end{aligned}$$

where e_1 is the smallest surplus of any coalition. A surplus e is defined as being the difference between the sum of the values allocated to the members of a coalition minus the characteristic function value of the coalition. Or in symbols this is

$$e = \sum_{i \in S} X_i - v(S).$$

The MPSX program for calculating the nucleolus is described below.

```
//   EXEC MPS
//CONTROL.SYSIN DD *
          PROGRAM
          INITIALZ
          MOVE(XDATA,'NUCLEO')
          MOVE(XPBNAME,'EXAMPLE')
          CONVERT
          BCDOUT
          SETUP('MAX')
          MOVE(XOBJ,'SURPLUS')
          MOVE(XRHS,'VALUE')
          PRIMAL
          SOLUTION
          EXIT
          PEND
/*
//PROBLEM.SYSIN DD *
NAME             NUCLEO
     (DATA DECK)
ENDATA
/*
```

The data deck includes three sections: rows, columns, and the rhs (right hand side). The rows section informs the computer of the type of row constraint; either greater than or equal to, less than or equal to, equal to, or an objective function. The columns section names the variables and gives the coefficient for each variable. The rhs section is the section in which the charactertistic function value of each row is given. Each row corresponds to one coalition.

The data deck for the first nucleolus run is shown below. The rows are called ROWA for the first row with only state AP, ROWT for the row with only state TN, etc.

```
..MPSX-V1M7..    EXECUTOR.   MPSX    RELEASE 1    MOD LEVEL 7

NAME              NUCLEO
ROWS
 N   SURPLUS
 G   ROWA
 G   ROWT
 G   ROWK
 G   ROWAT
 G   ROWAK
 G   ROWTK
 E   ROWATK
COLUMNS
    E             SURPLUS          1.00000     ROWA       -      1.00000
    E             ROWT      -      1.00000     ROWK       -      1.00000
    E             ROWAT     -      1.00000     ROWAK      -      1.00000
    E             ROWTK     -      1.00000
    A             ROWA             1.00000     ROWAT             1.00000
    A             ROWAK            1.00000     ROWATK            1.00000
    T             ROWT             1.00000     ROWAT             1.00000
    T             ROWTK            1.00000     ROWATK            1.00000
    K             ROWK             1.00000     ROWAK             1.00000
    K             ROWTK            1.00000     ROWATK            1.00000
RHS
    VALUE         ROWAT          210.00000     ROWAK           770.00000
    VALUE         ROWTK         1170.00000     ROWATK         1530.00000
ENDATA
```

The rows designation are N for objective function, G for greater than or equal to, E for equality, and L for less than or equal to.

The initial nucleolus program was run and e_1 was found to be 180 x 10^6 rupees. The rows section and the columns section from the first nucleolus run are shown below.

```
..MPSX-V1M7..    EXECUTOR.   MPSX    RELEASE 1    MOD LEVEL 7
```

```
SECTION 1 - ROWS

NUMBER  ...ROW..  AT   ...ACTIVITY...  SLACK ACTIVITY  ..LOWER LIMIT.  ..UPPER LIMIT.

     1  SURPLUS   BS      180.00000     180.00000-            NONE            NONE
     2  ROWA      LL          .              .                 .              NONE
     3  ROWT      BS      400.00000     400.00000-             .              NONE
     4  ROWK      BS      590.00000     590.00000-             .              NONE
     5  ROWAT     BS      580.00000     370.00000-        210.00000           NONE
     6  ROWAK     LL      770.00000          .            770.00000           NONE
     7  ROWTK     LL     1170.00000          .           1170.00000           NONE
     8  ROWATK    EQ     1530.00000          .           1530.00000      1530.00000

..MPSX-V1M7..    EXECUTOR.   MPSX    RELEASE 1    MOD LEVEL 7

SECTION 2 - COLUMNS

NUMBER   .COLUMN.   AT    ...ACTIVITY...   ..INPUT COST..

     9   E          BS       180.00000          1.00000
    10   A          BS       180.00000           .
    11   T          BS       580.00000           .
    12   K          BS       770.00000           .
```

Rows a, ak, and tk are at their lower limit in the rows section. For the next run of the nucleolus, that is finding e_2, we must fix one of the states' value. Since rowa and rowtk are there lower limit it must be that AP and the combination of TN and KM are fixed. The reason these two rows are fixed and not rowak is that the union of AP and TN-KM form the grand coalition, while AP-KM alone does not form the grand coalition without TN.

The second run of the nucleolus uses the following data deck. The remaining parts of the MPSX program remain the same.

```
..MPSX-V1M7..    EXECUTOR.   MPSX    RELEASE 1    MOD LEVEL 7

NAME             NUCLEO
ROWS
 N   SURPLUS
 E   ROWA
 G   ROWT
 G   ROWK
 G   ROWAT
 G   ROWAK
 E   ROWTK
 E   ROWATK
COLUMNS
    E            SURPLUS          1.00000     ROWT        -       1.00000
    E            ROWK     -       1.00000     ROWAT       -       1.00000
    E            ROWAK    -       1.00000
    A            ROWA             1.00000     ROWAT               1.00000
    A         ROWAK        1.00000    ROWATK             1.00000
    T         ROWT         1.00000    ROWAT              1.00000
    T         ROWTK        1.00000    ROWATK             1.00000
    K         ROWK         1.00000    ROWAK              1.00000
    K         ROWTK        1.00000    ROWATK             1.00000
RHS
    VALUE     ROWA       180.00000    ROWAT            210.00000
    VALUE     ROWAK      770.00000    ROWTK           1350.00000
    VALUE     ROWATK    1530.00000
ENDATA
```

As can be seen ROWA and ROWT are now equality rows and no longer have greater than or equal to constraints. Also the characteristic function values of AP and TN-KM are now v(s) + e_1 instead of v(s). The objective is to maximize e_2, where e_2 is the smallest surplus among the rows or coalitions without equality constraints.

The program was run and the value of e_2 found was 365 x 10^6 rupees. The columns and rows sections are shown below from this second nucleolus run.

```
..MPSX-V1M7..    EXECUTOR.   MPSX    RELEASE 1    MOD LEVEL 7

SECTION 1 - ROWS

NUMBER  ...ROW..  AT  ...ACTIVITY...  SLACK ACTIVITY  ..LOWER LIMIT.  ..UPPER LIMIT.

    1  SURPLUS   BS     365.00000       365.00000-          NONE            NONE
    2  ROWA      BS     180.00000            .          180.00000       180.00000
    3  ROWT      BS      30.00000        30.00000-           .              NONE
    4  ROWK      BS     590.00000       590.00000-           .              NONE
    5  ROWAT     LL     210.00000            .          210.00000           NONE
    6  ROWAK     LL     770.00000            .          770.00000           NONE
    7  ROWTK     EQ    1350.00000            .         1350.00000      1350.00000
    8  ROWATK    EQ    1530.00000            .         1530.00000      1530.00000

..MPSX-V1M7..    EXECUTOR.   MPSX    RELEASE 1    MOD LEVEL 7

SECTION 2 - COLUMNS

NUMBER   .COLUMN.    AT    ...ACTIVITY...    ..INPUT COST..

    9    E           BS       365.00000         1.00000
   10    A           BS       180.00000           .
   11    T           BS       395.00000           .
   12    K           BS       955.00000           .
```

Rows ROWAK and ROWAT are not at their lower limit. The union of AP-KM and AP-TN is the grand coalition so that TN and KM are fixed by the second nucleolus run. Note that this means of fixing the nucleolus value differs from the means in the first nucleolus run. The user must be careful when fixing values or costs for players to be used in subsequent nucleolus runs.

The nucleolus is now computed. The values assigned to the three players are

Andhra Pradesh	180,000,000 rupees
Tamil Nadu	395,000,000 rupees
Kerala-Mysore	955,000,000 rupees

The first nucleolus run took 1 second CPU time on the Amdahl 470 V/6 II as did the second nucleolus run. The total computing units defined as including the job submission service, job steps, HASP input and print service, CPU time, 3330 I/O operations, and 2k-minutes core was 0.2220 and 0.2191 for the first and second nucleolus runs, respectively.

The total then to compute the nucleolus of a three player, seven constraint game is 2 seconds CPU time and 0.4411 total computing units.

For sake of comparison the total time to compute the MCRS value of the same game is 1.98 seconds CPU time and 0.3846 total computing units. The comparison is seen to indicate that the MCRS and nucleolus are very similar in their computer usage for a small three person game.

The MCRS value will now be computed for the game involving the three states of India. The MCRS method in this value game will allocate values to the three states or players by the following equation.

$$x(i) = x(i)_{max} + \frac{x(i)_{min} - x(i)_{max}}{\sum_{i \in N} x(i)_{min} - \sum_{i \in N} x(i)_{max}} * [v(N) - \sum_{i \in N} x(i)_{max}] \qquad \text{(A-1)}$$

where $x(i)$ = the value allocated to player i,

$x(i)_{max}$ = the maximum value possible to player i in the core,

$x(i)_{min}$ = the minimum value possible to player i in the core, and

$v(n)$ = the characteristic function value of the grand coalition.

The MPSX program used to compute the MCRS values involves solving six linear programming problems; three maximization problems and three minimization problems. The program is the same as the nucleolus program except for some changes in the data deck and in the control program. The control program solves the six linear programming problems in one run. It is shown below.

```
..MPSX-V1M7..  CONTROL PROGRAM COMPILER.   MPSX   RELEASE 1   MOD LEVEL 7

          0001                  PROGRAM
          0002                  INITIALZ
          0096                  MOVE(XDATA,'NUCLEO')
          0097                  MOVE(XPBNAME,'EXAMPLE')
          0098                  CONVERT
          0099                  BCDOUT
```

```
0100          SETUP('MAX')
0101          MOVE(XDBJ,'OBJ')
0102          MOVE(XRHS,'VALUE')
0103          PRIMAL
0104          SOLUTION
0105          SETUP('MAX')
0106          MOVE(XOBJ,'OBJ1')
0107          MOVE(XRHS,'VALUE')
0108          PRIMAL
0109          SOLUTION
0110          SETUP('MAX')
0111          MOVE(XOBJ,'OBJ2')
0112          MOVE(XRHS,'VALUE')
0113          PRIMAL
0114          SOLUTION
0115          SETUP('MIN')
0116          MOVE(XOBJ','OBJ')
0117          MOVE(XRHS,'VALUE')
0118          PRIMAL
0119          SOLUTION
0120          SETUP('MIN')
0121          MOVE(XOBJ,'OBJ1')
0122          MOVE(XRHS','VALUE')
0123          PRIMAL
0124          SOLUTION
0125          SETUP('MIN')
0126          MOVE(XOBJ,'OBJ2')
0127          MOVE(XRHS,'VALUE')
0128          PRIMAL
0129          SOLUTION
0130          EXIT
0131          PEND
```

The six linear programming problems are to maximize x(AP), x(TN), and x(KM) or minimize x(AP), x(TN), and x(KM) subject to the following constraints.

$$AP \geq 0$$

$$TN \geq 0$$

$$KM \geq 0$$

$$AP + TN \geq 210$$

$$AP + KM \geq 770$$

$$TN + KM \geq 1170$$

$$AP + TN + KM = 1530$$

$$AP, KM, TN \geq 0$$

The data deck corresponding to these constraints is

```
..MPSX-V1M7..    EXECUTOR.   MPSX    RELEASE 1    MOD LEVEL 7

NAME            NUCLEO
ROWS
 N   OBJ
 N   OBJ1
 N   OBJ2
 G   ROWA
 G   ROWT
 G   ROWK
 G   ROWAT
 G   ROWAK
 G   ROWTK
 E   ROWATK
COLUMNS
    A           OBJ           1.00000     ROWA                1.00000
    A           ROWAT         1.00000     ROWAK               1.00000
    A           ROWATK        1.00000
    T           OBJ1          1.00000     ROWT                1.00000
    T           ROWAT         1.00000     ROWTK               1.00000
    T           ROWATK        1.00000
    K           OBJ2          1.00000     ROWK                1.00000
    K           ROWAK         1.00000     ROWTK               1.00000
    K           ROWATK        1.00000
RHS
    VALUE       ROWAT       210.00000     ROWAK             770.00000
    VALUE       ROWTK      1170.00000     ROWATK           1530.00000
ENDATA
```

As an illustration we will show the output from the programs to maximize and minimize x(AP). The rows and columns sections for the maximization of x(AP) are shown below.

```
..MPSX-V1M7..   EXECUTOR.   MPSX   RELEASE 1   MOD LEVEL 7

SECTION 1 - ROWS

NUMBER  ...ROW..  AT  ...ACTIVITY...  SLACK ACTIVITY  ..LOWER LIMIT.  ..UPPER LIMIT.

     1  OBJ       BS      360.00000      360.00000-           NONE            NONE
     2  OBJ1      BS      760.00000      760.00000-           NONE            NONE
     3  OBJ2      BS      410.00000      410.00000-           NONE            NONE
     4  ROWA      BS      360.00000      360.00000-            .              NONE
     5  ROWT      BS      760.00000      760.00000-            .              NONE
     6  ROWK      BS      410.00000      410.00000-            .              NONE
     7  ROWAT     BS     1120.00000      910.00000-      210.00000            NONE
     8  ROWAK     LL      770.00000          .           770.00000            NONE
     9  ROWTK     LL     1170.00000          .          1170.00000            NONE
    10  ROWATK    EQ     1530.00000          .          1530.00000      1530.00000

..MPSX-V1M7..    EXECUTOR.   MPSX    RELEASE 1    MOD LEVEL 7
```

```
SECTION 2 - COLUMNS

NUMBER   .COLUMN.   AT    ...ACTIVITY...    ..INPUT COST..

   11    A          BS      360.00000          1.00000
   12    T          BS      760.00000           .
   13    K          BS      410.00000           .
```

The rows and columns sections for the minimization of x(AP) are shown below.

```
..MPSX-V1M7..    EXECUTOR.   MPSX    RELEASE 1    MOD LEVEL 7

SECTION 1 - ROWS

NUMBER  ...ROW..  AT  ...ACTIVITY...  SLACK ACTIVITY  ..LOWER LIMIT.  ..UPPER LIMIT.

    1   OBJ       BS         .              .                NONE            NONE
    2   OBJ1      BS     760.00000      760.00000-           NONE            NONE
    3   OBJ2      BS     770.00000      770.00000-           NONE            NONE
    4   ROWA      BS         .              .                 .              NONE
    5   ROWT      BS     760.00000      760.00000-            .              NONE
    6   ROWK      BS     770.00000      770.00000-            .              NONE
    7   ROWAT     BS     760.00000      550.00000-        210.00000          NONE
    8   ROWAK     LL     770.00000          .             770.00000          NONE
    9   ROWTK     BS    1530.00000      360.00000-       1170.00000          NONE
   10   ROWATK    EQ    1530.00000          .            1530.00000      1530.00000

..MPSX-V1M7..    EXECUTOR.   MPSX    RELEASE 1    MOD LEVEL 7

SECTION 2 - COLUMNS

NUMBER   .COLUMN.   AT    ...ACTIVITY...    ..INPUT COST..

   11    A          LL         .               1.00000
   12    T          BS      760.00000           .
   13    K          BS      770.00000           .
```

The results of the six linear programming problems are

x(AP)max...........	360
x(TN)max...........	760
x(KM)max..........	1320
x(AP)min.............	0
x(TN)min.............	0
x(KM)min...........	410 with all values x 10^6 rupees.

Using these maximum and minimum values the MCRS solution is calculated to be the following:

Andhra Pradesh...........	198,600,000 rupees
Tamil Nadu...............	419,400,000 rupees
Kerala-Mysore............	912,000,000 rupees

This differs from the nucleolus in that the MCRS does not reward Kerala-Mysore quite as much as the nucleolus. The comparison between the CPU times and total computing units of the MCRS and the nucleolus have already been discussed.

In conclusion, the linear programming solution of the nucleolus and MCRS is easy and straightforward. The difficult part of using these game theoretical concepts is in calculating the characteristic function costs or values of a coalition.